Math 390 Thursday, April 22 Determinants

$A = (a_{ij})$ ($A+B = C = (c_{ij})$

$a_{ij} + b_{ij} = c_{ij}$

$[A+B]_{ij} = [A]_{ij} + [B]_{ij}$)

Fri BYOB

Sun Nite, 11:59 PM

Final Paper

20%

$$\det(A) = \sum_{\sigma \in S_n} \operatorname{sign}(\sigma)\, a_{1\sigma(1)}\, a_{2\sigma(2)} \cdots a_{n\sigma(n)}$$

σ - permutation (1-1 onto function $\{1,2,\ldots,n\} \to \{1,2,\ldots,n\}$)

S_n - <u>all</u> possible permutation of $\{1,2,\ldots,n\}$, there are $n!$ of these

$\operatorname{sign}(\sigma) = +1$ or -1

Every permutation can be brought to its "natural order" through a number of "swaps", technical <u>transpositions</u>.

$$\sigma = \overset{1}{2}\,\overset{2}{4}\,\overset{3}{1}\,\overset{4}{3} \qquad 2413 \to 1423 \to 1243 \to 1234$$

3 swaps $\Rightarrow$ $\mathrm{sign}(\sigma) = (-1)^3 = -1$

Doob works left to right getting each position in order

Sorting Algorithms

- Worse than Bubble sort

$$2413 \to 2143 \to 2134 \to 1234$$

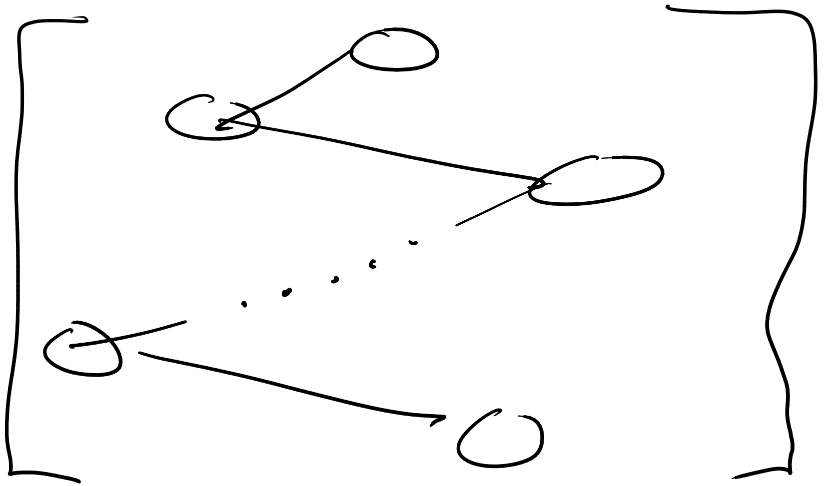
3 swaps, same <u>parity</u> as

<u>Defn</u> $\mathrm{sign}(\sigma) = (-1)^s$ where $s = \#$ swaps (transpositions) to bring σ to natural order

Theorem $\mathrm{sign}(\sigma)$ is well-defined.

Proof Math 490.

One summand of $\det(A)$



Ex $A = \begin{bmatrix} 2 & 1 & 2 \\ -1 & 3 & -2 \\ -1 & 2 & 1 \end{bmatrix}$ $\det A$?

σ	$\operatorname{sgn}(\sigma)$	product entries	signed
1 2 3	1	$2 \cdot 3 \cdot 1 = 6$	6
$\overset{1}{1}\ \overset{2}{3}\ \overset{3}{2}$	-1	$2(-2)(2) = -8$	8
2 1 3	-1	$1(-1)1 = -1$	1
2 3 1	1	$1(-2)(-1) = 2$	2
3 1 2	1	$2(-1)(2) = -4$	-4
3 2 1	-1	$2(3)(-1) = -6$	6
			19

$a_{1\sigma(1)} a_{2\sigma(2)} a_{3\sigma(3)}$
$= a_{11} a_{23} a_{32}$

50% -1
50% 1 ✓

$$\begin{bmatrix} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{bmatrix}$$

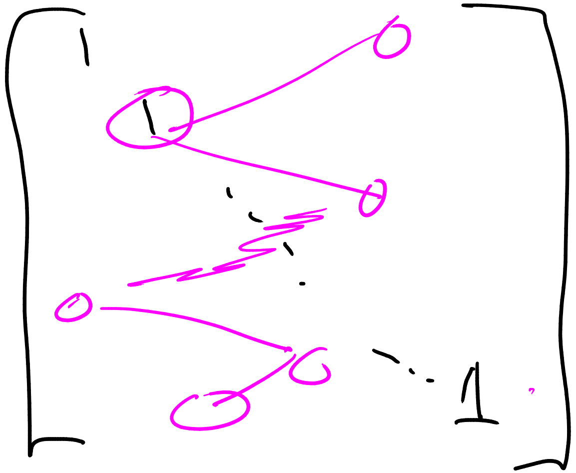
$-$

$+$

Fact $\det(I_n) = 1$

Proof

$$\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & \ddots & \\ & & & & & 1 \end{bmatrix} + $$



$$= 1$$

$\sigma = 12345 \cdots n$

even, contribution is $(1)\, 1 \cdot 1 \cdots 1$

$\uparrow$

sign σ

$\sigma \neq 12 \cdots n$

contribution is

$\operatorname{sign}(\sigma) = 0 \cdot 0 \cdot 1 \cdot 0 \cdots 0 \cdot 0$