Math 390 Friday, April 9

Least-Squares

Mon - Least-Squares

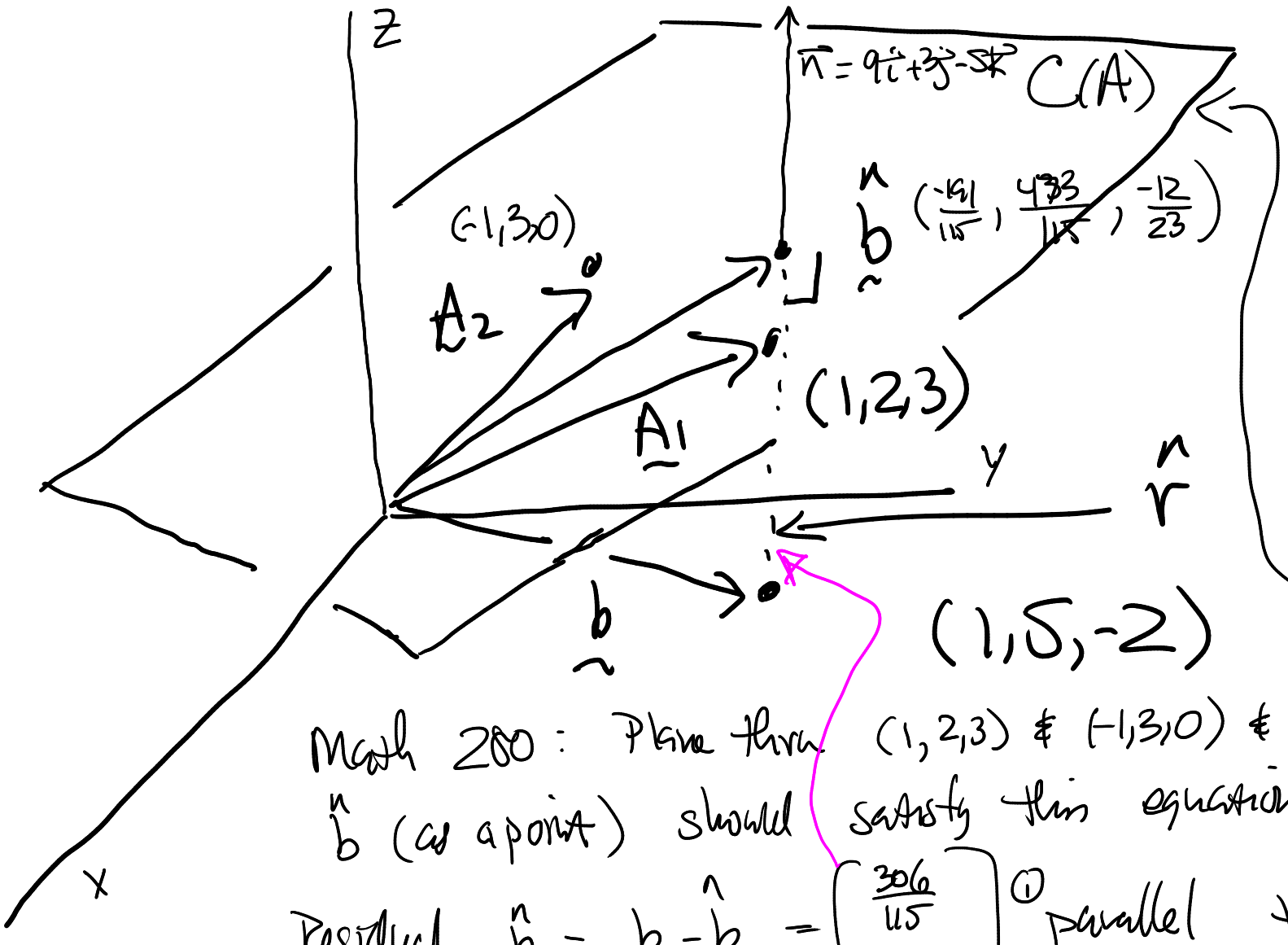
Tue - Roblem Session

Projects (40%)

- 1 week Draft
- 1 week Final
- 1 week Presentations

Beamer

"Technically Speaking"



$$\begin{bmatrix} 1 & -1 \\ 2 & 3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ -2 \end{bmatrix}$$

A_1 A_2 $\underset{\sim}{b}$

$\dim C(A) = 2 \leq 3 = \dim(\mathbb{R}^3)$

$$\hat{\underset{\sim}{b}} = A\hat{\underset{\sim}{x}} = \begin{bmatrix} \frac{-191}{115} \\ \frac{473}{115} \\ -12/23 \end{bmatrix}$$

Math 200: Plane thru (1,2,3) & (-1,3,0) & (0,0,0) : $9x + 3y - 5z = 0$

$\hat{b}$ (as a point) should satisfy this equation

normal to plane

Residual $\hat{r} = \underset{\sim}{b} - \hat{\underset{\sim}{b}} = \begin{bmatrix} \frac{306}{115} \\ \frac{102}{115} \\ -34/23 \end{bmatrix}$ ① parallel to $\vec{n} = 9\vec{i} + 3\vec{j} - 5\vec{k}$

② orthogonal to $\underset{\sim}{A_1}$ & $\underset{\sim}{A_2}$

Model of construction activity & interest rates

$$R = aC + b \qquad \text{(linear model)}$$

R: interest rates

C: construction permits (per capita)

30 day period, 60 years

Data (c_i, r_i) $\quad 1 \le i \le 60 \cdot 12 = 720$

$$r_i = a c_i + b \qquad 1 \le i \le 720$$

How likely is it that $\underset{\sim}{b} \in C(A)$?

$$\begin{bmatrix} c_1 & 1 \\ c_2 & 1 \\ c_3 & 1 \\ \vdots & \\ c_{720} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{720} \end{bmatrix}$$

A — solve for parameters — $\underset{\sim}{b}$

Normal Equation $A^* A \underset{\sim}{x} = A^* \underset{\sim}{b}$

$$A^* A = \begin{bmatrix} c_1 & \cdots & c_{720} \\ 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} c_1 & 1 \\ \vdots & \vdots \\ c_{720} & 1 \end{bmatrix} = \begin{bmatrix} \sum c_i^2 & \sum c_i \\ \sum c_i & 720 \end{bmatrix}$$

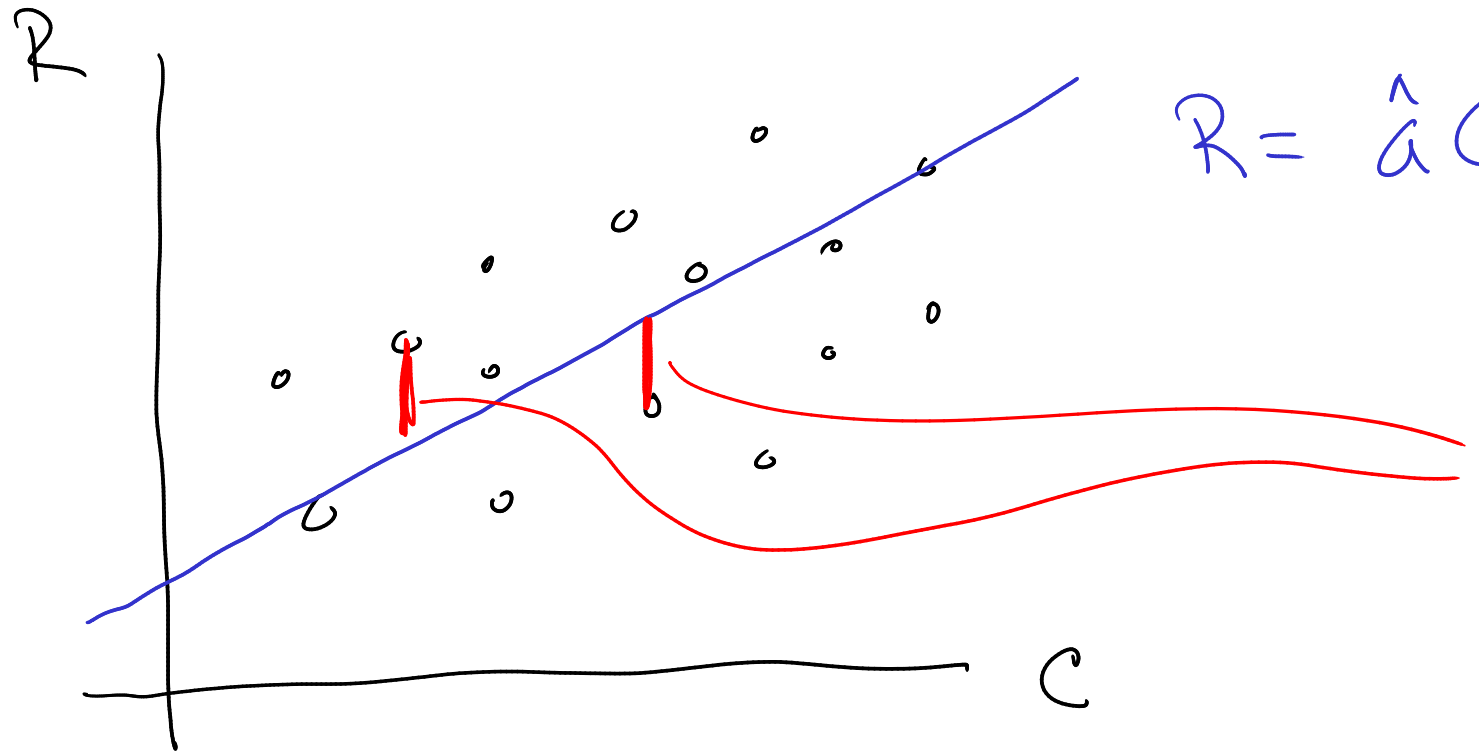
$$A^* \underset{\wedge}{b} = \begin{bmatrix} c_1 & \cdots & c_{720} \\ 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_{720} \end{bmatrix} = \begin{bmatrix} \sum c_i r_i \\ \sum r_i \end{bmatrix}$$

Solution $\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \hat{x} = \begin{bmatrix} \sum c_i^2 & \sum c_i \\ \sum c_i & 720 \end{bmatrix}^{-1} \begin{bmatrix} \sum c_i r_i \\ \sum r_i \end{bmatrix}$

$$= \frac{1}{720 \sum c_i^2 - \left(\sum c_i\right)^2} \begin{bmatrix} 720 & -\sum c_i \\ -\sum c_i & \sum c_i^2 \end{bmatrix} \begin{bmatrix} \sum c_i r_i \\ \sum r_i \end{bmatrix}$$

$$= \frac{1}{720 \sum c_i^2 - \left(\sum c_i\right)^2} \begin{bmatrix} 720 \sum c_i r_i - \sum c_i \sum r_i \\ -\sum c_i \sum c_i r_i + \sum c_i^2 \sum r_i \end{bmatrix} \begin{matrix} \hat{a} \\ \hat{b} \end{matrix}$$

Linear Regression
Math 360



$$R = \hat{a}C + \hat{b}$$

+wo components of $\underset{\sim}{r} = \underset{\sim}{b} - \underset{\sim}{\hat{b}}$