Math 390 Monday, April 5 Problem Session

Jordan Canonical Form

(Rational Canonical Form)

LU, QR, SVD, Schur, Cholesky

(Big 5)

Sage: similar to worksheets

The Exam 2

Laptop

Thu Least Squares

1.3.3

$U = \left\langle \left\{ \begin{bmatrix} 1 \\ -6 \\ -8 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ -7 \end{bmatrix} \right\} \right\rangle$: W so that $\mathbb{C}^3 = U \oplus W$

$\dim(U) = 2 \Rightarrow \dim(W) = 1$

$\underset{\sim}{w} \neq \begin{bmatrix} 1 \\ -6 \\ -8 \end{bmatrix} + \begin{bmatrix} 1 \\ -5 \\ -7 \end{bmatrix} = \begin{bmatrix} 2 \\ -11 \\ -15 \end{bmatrix}$, choose $\underset{\sim}{w} = \begin{bmatrix} 2 \\ -11 \\ 847 \end{bmatrix} \notin U$

$W = \langle \{ \underset{\sim}{w} \} \rangle$

$U^\perp$ - orthogonal complement

$U = C(A) \quad A = \begin{bmatrix} 1 & 1 \\ -6 & -5 \\ -8 & -7 \end{bmatrix} \Rightarrow U^\perp = N(A^*) = N\left(\begin{bmatrix} 1 & -6 & -8 \\ 1 & -5 & -7 \end{bmatrix} \right)$

$\begin{bmatrix} 1 & -6 & -8 \\ 1 & -5 & -7 \end{bmatrix} \to \begin{bmatrix} 1 & -6 & -8 \\ 0 & 1 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix} = \left\langle \left\{ \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\} \right\rangle$

2.4.2

$A = L^* L$ (reminiscent of LU decomposition)

$A = L_1^* L_1 \quad \& \quad A = L_2^* L_2$

$L_1^* L_1 = L_2^* L_2$

$L_2 L_1^* L_1 = L_2$

$L_2 L_1^* = L_2 L_1^*$

$L_2 L_1^* = L_2 A A^{-1} L_1^*$

$= L_2 \boxed{L_2^* L_2} (L_1^* L_1)^{-1} L_1^*$

$= L_2 L_1^{-1} (L_1^*)^{-1} L_1^*$

$= L_2 L_1^* L_1 L_1^*$

$= L_2 L_1^*$

$$L_2 L_2^{-1} = \cancel{L_2 A A^{-1} L_2^{-1}}$$

$$\cancel{= L_2 L}$$

$$L_2^{-1} L_1 = \cancel{L_2^{-1} A A^{-1} L_1}$$

$$\cancel{= L_2^{-1}}$$

$$\underbrace{L_2 L_1^{-1}}_{\text{lower triangular}} = L_2 A^{-1} A L_1^{-1} = L_2 (L_2^* L_2)^{-1} (L_1^* L_1) L_1^{-1} = L_2 L_2^{-1} (L_2^*)^{-1} L_1^* L_1 L_1^{-1}$$

$$= \underbrace{(L_2^*)^{-1} L_1^*}_{\text{upper triangular}}$$

$\Rightarrow$ these matrices are diagonal

~~diagonal entries of L_2 & L_1 are the same~~

diagonal entries of $L_2 L_1^{-1}$ are all 1

$\Rightarrow L_2 L_1^{-1} = I_n \Rightarrow L_2 = L_1$

diagonal entry of

$L_2 L_1^{-1} = l_2 l_1^{-1}$

$(L_2^*)^{-1} L_1^* = \bar{l}_2^{-1} \bar{l}_1$

reciprocals & equal

$\Rightarrow l_2 l_1^{-1} = 1$

$l_2 l_1^{-1} = \bar{l}_2^{-1} \bar{l}_1$

$l_2 \bar{l}_2 = l_1 \bar{l}_1$

Jack's approach

$-1?$

$$I = L_1^{-1} L_2 L_2^* L_1^* = (L_1^{-1} L_2)(L_1^{-1} L_2)^*$$

$$\Rightarrow L_1^{-1} L_2 = \left((L_1^{-1} L_2)^{-1} \right)^* \Rightarrow L^{-1} L_2 \text{ diagonal}, \ I = (L^{-1} L_2)^2 \text{ to get}$$

1's on diagonal