Math 390 Thursday, April 1 Cholesky Factorization

Fri – Foreign Country

– ?

Mon – Problem Session

Tue – Exam 2 (Factorizations)

A – Hermitian (real entries on diagonal)

– positive definite

$\langle \underset{\sim}{x}, A\underset{\sim}{x} \rangle > 0$ for all $\underset{\sim}{x} \neq \underset{\sim}{0}$

$\iff$ positive eigenvalues

then ~~$A = U^* U$, where U is upper-triangular~~

<u>OR</u> $A = LL^*$ where L is lower-triangular ← common

Overview: like LU decomposition, but do row operations on the left, and similar (adjoint) column operations on right.

Computing Cholesky

$\langle \underset{\sim}{e_1}, A\underset{\sim}{e_1} \rangle = \langle \underset{\sim}{e_1},$ first column of $A \rangle =$ upper-left entry of A

> 0 (and real)

Write

$$A = \left[\begin{array}{c|c} a & \underset{\sim}{y}^* \\ \hline \underset{\sim}{y} & B \end{array}\right] ; \quad a > 0$$

then ← lower triangular

$$\left[\begin{array}{c|c} \sqrt{a} & \underset{\sim}{0}^* \\ \hline \frac{1}{\sqrt{a}}\underset{\sim}{y} & I \end{array}\right] \left[\begin{array}{c|c} 1 & \underset{\sim}{0}^* \\ \hline \underset{\sim}{0} & B - \frac{1}{a}\underset{\sim}{y}\underset{\sim}{y}^* \end{array}\right] \left[\begin{array}{c|c} \sqrt{a} & \frac{1}{\sqrt{a}}\underset{\sim}{y}^* \\ \hline \underset{\sim}{0} & I \end{array}\right]$$

take an adjoint

A_1

need square roots

need $a > 0$, $a \neq 0$

$$= \left[\begin{array}{c|c} \sqrt{a}\cdot 1 + \underset{\sim}{0}^* \underset{\sim}{0} & \sqrt{a}\,\underset{\sim}{0}^* + \underset{\sim}{0}^*(B - \frac{1}{a}yy^*) \\ \hline \frac{1}{\sqrt{a}}\underset{\sim}{y}\cdot 1 & \frac{1}{\sqrt{a}}\underset{\sim}{y}\cdot 0^* + I(B - \frac{1}{a}yy^*) \end{array}\right] \left[\begin{array}{c|c} \sqrt{a} & \frac{1}{\sqrt{a}}y^* \\ \hline \underset{\sim}{0} & I \end{array}\right]$$

$$= \left[\begin{array}{c|c} \sqrt{a} & \underset{\sim}{0} \\ \hline \frac{1}{\sqrt{a}}\underset{\sim}{y} & B - \frac{1}{a}yy^* \end{array}\right] \left[\begin{array}{c|c} \sqrt{a} & \frac{1}{\sqrt{a}}y^* \\ \hline \underset{\sim}{0} & I \end{array}\right]$$

$$= \left[\begin{array}{c|c} a & \underset{\sim}{y}^* \\ \hline \underset{\sim}{y} & \left(\frac{1}{\sqrt{a}}\underset{\sim}{y}\right)\left(\frac{1}{\sqrt{a}}y^*\right) + B - \frac{1}{a}yy^* \end{array}\right] = \begin{bmatrix} a & y^* \\ \underset{\sim}{y} & B \end{bmatrix} = A$$

$$A = L_1 \, A_1 \, L_1^* = L_1 L_2 \left[\begin{array}{c|cc} 1 & 0 & \\ \hline 0 & 1 & 0 \\ & 0 & \ddots \end{array}\right] L_2^* L_1^* = L I L^*$$

$$= L L^*$$

A_1: 1 in upper left corner, 0's in first row, first column