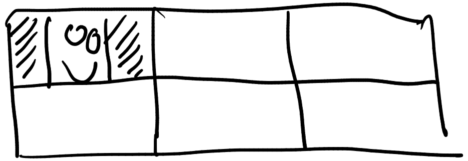
Math 390 Monday, March 22

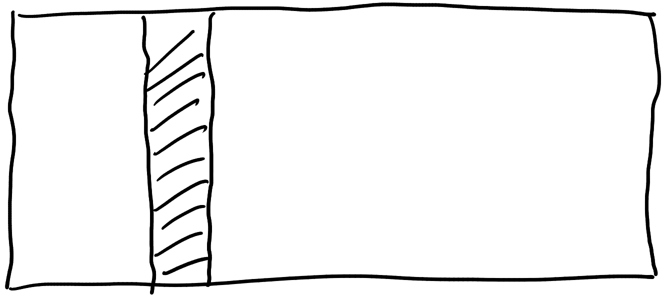
SVD, Orthonormal Diagonalization / Schur Form

Image Compression



$k=1$

$s_1 y_1 x_1^*$



Tue - Virtual

Thu - FCLA OD

__Projects__ Applications of SVD

- Netflix Prize (\$1 million)
- Google Page Rank

Visualization of SVD

$$A = U S V^*$$

U: isometry "rotation"

S: "stretch" contract, reflect

V^*: unitary "rotation" __isometry__

$\langle U\underset{\sim}{x}, U\underset{\sim}{y}\rangle = \langle \underset{\sim}{x}, \underset{\sim}{y}\rangle$ preserves angles

$\underset{\sim}{u}\cdot\underset{\sim}{v} = \|\underset{\sim}{u}\|\,\|\underset{\sim}{v}\|\cos\theta$

$\langle U\underset{\sim}{x}, U\underset{\sim}{x}\rangle = \langle \underset{\sim}{x}, \underset{\sim}{x}\rangle$

$\|U\underset{\sim}{x}\| = \|\underset{\sim}{x}\|$ preserve lengths

Columns of U & V are "axes" for codomain & domain (respectively)

Start $\quad A\underset{\sim}{w} = \underset{\sim}{y} \quad$ A $m \times n$

basis in multivariate

$\vec{i}, \vec{j}, \vec{k}$

$\underset{\sim}{e}_1, \underset{\sim}{e}_2, \underset{\sim}{e}_3$

$\underset{\sim}{w} = V\underset{\sim}{w}'$ $\quad$ w' coordinate vector w relative to orthonormal basis V $\quad$ $w' = V^* \underset{\sim}{w}$

$V^* = V^{-1}$

$\underset{\sim}{y} = Uy'$ $\quad$ y' coordinate vector for y relative to orthonormal basis U $\quad$ $y' = U^* \underset{\sim}{y}$

Then

$$y' = U^* \underset{\sim}{y} = U^* \underline{A} \underset{\sim}{w} = U^* \underline{USV^*} \underset{\sim}{w} = Sw'$$

$A\underset{\sim}{w} = \underset{\sim}{y}$

standard bases

$\underset{\sim}{e}_1, \underset{\sim}{e}_2, \ldots$ (axes)

$Sw' = y'$

basis V (axes) $\quad$ basis U (axes)

extremely simple matrix/linear transformation representation