Math 290 Monday, May 3 Problem Session

Final Exam

Monday, May 10

4th 395, 8-11 AM

Two-hour material

7 pages

Comprehensive, proofs +

office hours - by appointment

Chapter R Exam

- matrix representation — a skill
  "on sight"
- No proofs.

CB. T15 $T: V \to V$ T injective $\iff$ $\lambda = 0$ not an eigenvalue of T

T not injective $\iff$ $\lambda = 0$ is an eigenvalue

($\Rightarrow$) T not injective

$\Rightarrow$ $\underset{\sim}{x} \neq \underset{\sim}{y}$, $T(\underset{\sim}{x}) = T(\underset{\sim}{y})$

there exists

$T(\underset{\sim}{x}) = T(\underset{\sim}{y}) \longrightarrow T(\underset{\sim}{x}) - T(\underset{\sim}{y}) = \underset{\sim}{0} \to T(\underset{\sim}{x} - \underset{\sim}{y}) = \underset{\sim}{0}$

$\longrightarrow$ ① $T(\underset{\sim}{x} - \underset{\sim}{y}) = 0\,(\underset{\sim}{x} - \underset{\sim}{y})$ ②

$\neq \underset{\sim}{0}$

0 eigenvalue, for eigenvector $\underset{\sim}{x} - \underset{\sim}{y}$.

($\Leftarrow$) $\lambda = 0$ is an eigenvalue, so let $\underset{\sim}{x} \neq \underset{\sim}{0}$ that is eigenvector for $\lambda = 0$

$T(\underset{\sim}{x}) = \lambda \underset{\sim}{x} = 0\,\underset{\sim}{x} = \underset{\sim}{0} = T(\underset{\sim}{0})$

LTTZZ

So T is not injective. $\underset{\sim}{x} \neq \underset{\sim}{0}$.

MR. C51

$L: M_{22} \to P_2 \qquad L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a+2b+4c+d) + (3a+c-2d)x + (-a+b+3c+2d)x^2$

$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\} \qquad C = \{1, x, x^2\}$

K(L) ?

"on sight"

$$M^L_{B,C} = \begin{bmatrix} 1 & 2 & 4 & 1 \\ 3 & 0 & 1 & -2 \\ -1 & 1 & 3 & 3 \end{bmatrix}$$

FICTIONAL

$K(L) \underset{(\rho_B)}{\cong} N(M^L_{B,C}) = \left\langle \left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 0 \\ 1 \end{bmatrix} \right\} \right\rangle$

ρ_B^{-1}

$K(L) = \left\langle \left\{ \begin{bmatrix} 3 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \right\} \right\rangle$