Math 290 Friday, April 23 Section MR

$T : U \to V$ Bases B, C;

$B = \{\underset{\sim}{u_1}, \underset{\sim}{u_2}, \dots, \underset{\sim}{u_n}\}$

$$M^T_{B,C} = \left[\rho_C(T(\underset{\sim}{u_1})) \middle| \cdots \middle| \rho_C(T(\underset{\sim}{u_n}))\right]$$

Mon - Problems (LT)

Tue - Exam LT

Thu - Problems (VR, MR)

Fri - CB

__Ex__ $T : M_{22} \to P_2$

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (2a + b + 3c - 2d) + (5a + 3b + 7c - 4d)x + (a + b + c)x^2$$

$$B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$D = \{1, x, x^2\}$$

$$C = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} \right\}$$

$$E = \{1, 1+x, 1+x+x^2\}$$

$\underline{M^T_{C,E}}$

$$\rho_E\left(T\left(\begin{bmatrix}1 & -2\\ 1 & -1\end{bmatrix}\right)\right) = \rho_E\left(5 + 10x + 0x^2\right)$$

$$= \rho_E\left((-5)(1) + 10(1+x) + (0)(1+x+x^2)\right) = \begin{bmatrix}-5\\ 10\\ 0\end{bmatrix}$$

3 more

$$M^T_{C,E} = \begin{bmatrix}-5 & -12 & -5 & -12\\ 10 & 20 & 12 & 16\\ 0 & 0 & -1 & 4\end{bmatrix}$$

How are these related?

Section CB

$\underline{M^T_{B,D}}$

$$\rho_D\left(T\left(\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\right)\right) = \rho_D\left(2 + 5x + x^2\right) = \begin{bmatrix}2\\ 5\\ 1\end{bmatrix}$$

3 more

$$M^T_{B,D} = \begin{bmatrix}2 & 1 & 3 & -2\\ 5 & 3 & 7 & -4\\ 1 & 1 & 1 & 1\end{bmatrix}$$

"on sight"

Ex $T: P_2 \to P_2 \quad T(a+bx+cx^2) = (5a+2b+2c)+(-10a-3b-2c)x + (6a+2b+c)x^2$

$$B = C = \{1-2x+x^2,\ 1-3x+x^2,\ x-x^2\}$$

$$M^T_{B,B}$$

$$\rho_B(T(1-2x+x^2)) = \rho_B(3-6x+3x^2) = \rho_B\big(3(1-2x+x^2)+0(1-3x+x^2)+0(x-x^2)\big)$$
$$= \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

$$\rho_B(T(1-3x+x^2)) = \rho_B(1-3x+x^2) = \rho_B\big(0(1-2x+x^2)+1(1-3x+x^2)+0(x-x^2)\big) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\rho_B(T(x-x^2)) = \rho_B(-x+x^2) = \rho_B\big(0(1-2x+x^2)+0(1+3x-x^2)+(-1)(x-x^2)\big) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

$$M^T_{B,B} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

FTMR $\quad T : U \to V \quad$ Bases B & C

mvp, doing T

$$\rho_C(T(\underset{\sim}{u})) = M^T_{B,C}\,\rho_B(\underset{\sim}{u})$$

doing T $\qquad$ vector equality

$$T(\underset{\sim}{u}) = \rho_C^{-1}\big(M^T_{B,C}\,\rho_B(\underset{\sim}{u})\big)$$

③ $\qquad$ mvp ② $\qquad$ ①

$$\begin{tikzcd}
\underset{\sim}{u} \arrow[r, "T"] \arrow[d, "\rho_B"'] & T(\underset{\sim}{u}) \arrow[d, shift left, "\rho_C"] \\
\rho_B(\underset{\sim}{u}) \arrow[r, "M^T_{B,C}"'] & M^T_{B,C}\,\rho_B(\underset{\sim}{u}) \arrow[u, shift left, "\rho_C^{-1}"]
\end{tikzcd}$$

$\rho_C(T(\underset{\sim}{u}))$

$M^T_{B,C}\,\rho_B(\underset{\sim}{u})$ $\qquad$ equal

$$
\begin{tikzcd}
& U \arrow[r, "T"] \arrow[dd, "\rho_B"] \arrow[dl, dash, dashed] & V \arrow[dd, "\rho_C"] \arrow[dr, dash, dashed] & \\
\mathcal{K}(T) \arrow[dd, "\rho_B|_{\mathcal{K}(T)}"] & & & \mathcal{R}(T) \arrow[dd, "\rho_C|_{\mathcal{R}(T)}"] \\
& \mathbb{C}^n \arrow[r, "M^T_{B,C}"] \arrow[dl, dash, dashed] & \mathbb{C}^m \arrow[dr, dash, dashed] & \\
\mathcal{N}(M^T_{B,C}) & & & \mathcal{C}(M^T_{B,C})
\end{tikzcd}
$$

KRI

$$M^{T^{-1}}_{C,B} = \left(M^T_{B,C}\right)^{-1}$$