Math 290 Thursday, April 22 Section VR

Fri – MR

BYOB – Free Choice

Mon – Problems

Tue – Exam LT

LT-1 : <u>no closure</u>!

LT-2 : style rewrite

<u>Ex</u> $\mathbb{C}^3$ $B = \left\{ \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix} \right\}$

B <u>is</u> a basis

Consider $\underset{\sim}{x} = \begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix}$

Express ("write") $\underset{\sim}{x}$ as a linear combination of vectors in B, <u>uniquely</u>. Theorem VRRB

$$\begin{bmatrix} -12 \\ 9 \\ 16 \end{bmatrix} = 2\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix} + (-1)\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix} + 2\begin{bmatrix} -2 \\ 3 \\ 7 \end{bmatrix}$$

solve relevant system, unique solution

then $\rho_B(\underset{\sim}{x}) = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$

$\underline{Ex}$ M_{22} $C = \left\{ \begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix}, \begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix} \right\}$ basis

$$\underset{\sim}{x} = \begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix} = 2\begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix} + 0\begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} + (-1)\begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix} + 4\begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix}$$

set up linear combo w/ unknown scalars a_1, a_2, a_3, a_4

Solve 4x4 system, unique solution, nonsingular coefficient matrix

$$\rho_C(\underset{\sim}{x}) = \begin{bmatrix} 2 \\ 0 \\ -1 \\ 4 \end{bmatrix}$$

"Same" matrix

$$D = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 8 & 7 \\ 23 & 21 \end{bmatrix} = 8\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 7\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + 23\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + 21\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{so } \rho_D(\underset{\sim}{x}) = \begin{bmatrix} 8 \\ 7 \\ 23 \\ 21 \end{bmatrix}$$

"on sight"

"coordinatization"

Theorem VRILT ρ_B is an invertible linear transformation.

Ex $\rho_C : M_{22} \to \mathbb{C}^4$ is invertible, hence isomorphism.

So we can say M_{22} & $\mathbb{C}^4$ are isomorphic

Fact $\dim(V) = n \implies V \cong \mathbb{C}^n$

"isomorphic"

Fact $\dim(U) = \dim(V) \implies U \cong V$

Ex C, M_{22} $\rho_C^{-1} : \mathbb{C}^4 \to M_{22}$

$$\rho_C^{-1}\left(\begin{bmatrix} 1 \\ 2 \\ -1 \\ 3 \end{bmatrix}\right) = 1\begin{bmatrix} -2 & 4 \\ 3 & 3 \end{bmatrix} + 2\begin{bmatrix} 0 & -2 \\ 3 & -1 \end{bmatrix} + (-1)\begin{bmatrix} 0 & 5 \\ 3 & 5 \end{bmatrix} + 3\begin{bmatrix} 3 & 1 \\ 5 & 5 \end{bmatrix} = \begin{bmatrix} 7 & \\ & \end{bmatrix}$$

"uncoordinization"

Section MR

matrix representations of linear transformations

$T: U \to V$ $\qquad$ B, C are bases of U & V (respectively)

$B = \{ \underset{\sim}{u_1}, \underset{\sim}{u_2}, \ldots, \underset{\sim}{u_n} \}$

$$M^T_{B,C} = \left[\rho_C(T(\underset{\sim}{u_1})) \middle| \rho_C(T(\underset{\sim}{u_2})) \middle| \cdots \middle| \rho_C(T(\underset{\sim}{u_n})) \right]$$