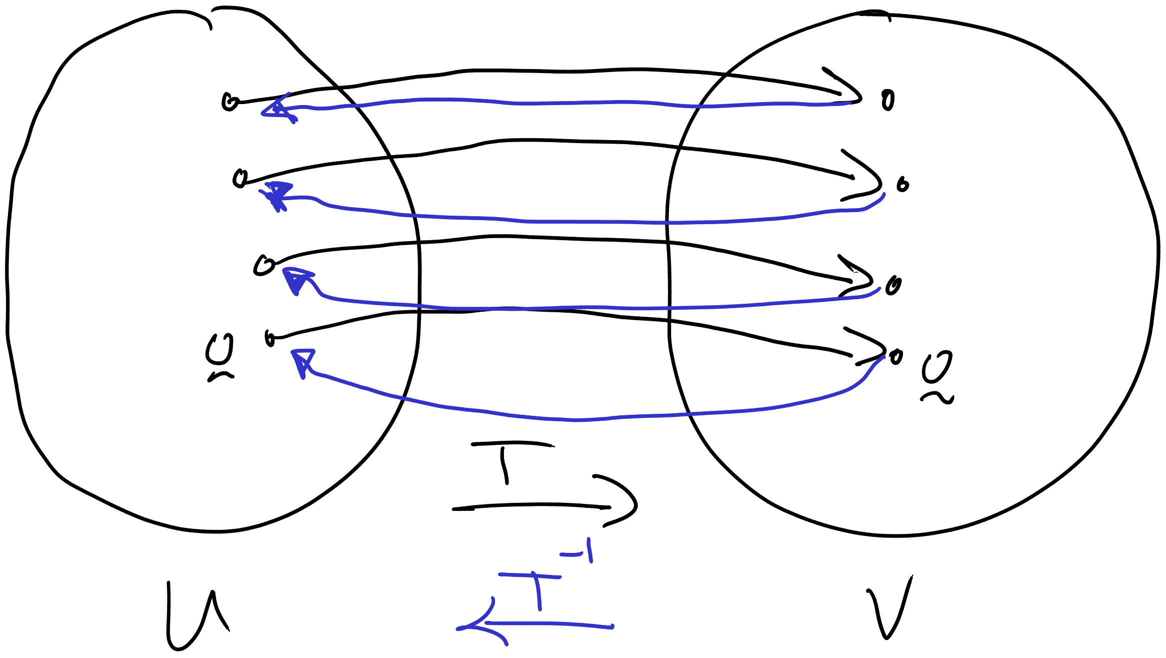
Math 290 Monday, April 19 Section IVLT

Invertible Linear Transformations

- injective & surjective (ILTIS)




invertible: injective & surjective

The Problem Session

Writing LT

Thu - VR

Fri - MR

BYOB - Free Choice

Each $\underset{\sim}{v} \in V$ has $T^{-1}(\underset{\sim}{v})$ is a singleton

Ex $T: P_3 \Rightarrow M_{22}$ $\quad T(a+bx+cx^2+dx^3) = \begin{bmatrix} a+b+c+d & 2a+3b+4c-d \\ -a-b+d & 2a+3b+5c+2d \end{bmatrix}$

$T^{-1}$? ( $T$ is injective & surjective.)

Grab a basis of codomain, $M_{22}$, $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

Compute pre-images.

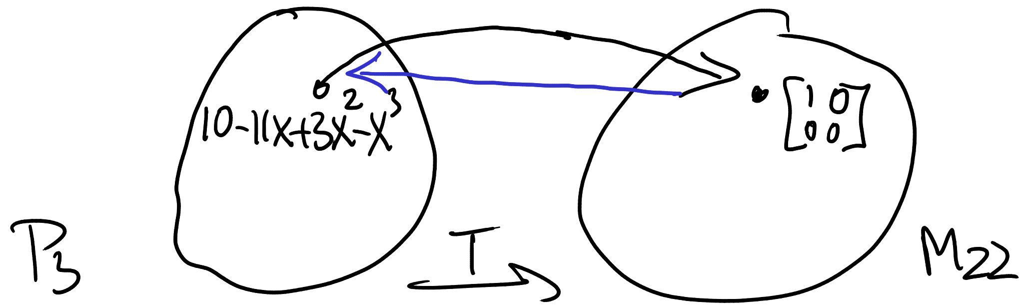
$T^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right)$? $\quad T(a+bx+cx^2+dx^3) = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} a+b+c+d=1 \\ 2a+3b+4c-d=0 \\ -a-b+d=0 \\ 2a+3b+5c+2d=0 \end{array} \overset{\text{RREF}}{\Rightarrow} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 & -11 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$

$T^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) = \{10-11x+3x^2-x^3\}$

$T^{-1}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right)$? $\quad T(a+bx+cx^2+dx^3) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \Rightarrow \begin{array}{l} a+b+c+d=0 \\ 2a+3b+4c-d=1 \\ -a-b+d=0 \\ 2a+3b+5c+2d=0 \end{array} \overset{\text{RREF}}{\Rightarrow} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & -6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$

$T^{-1}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) = \{5-6x+2x^2+x^3\}$

$T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) = \{7-9x+3x^2-x^3\}$ $\quad T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) = \{-6+7x-2x^2+x^3\}$



$$T^{-1}: M_{22} \to P_3 \qquad T^{-1}\left(\begin{bmatrix} e & f \\ g & h \end{bmatrix}\right) = T^{-1}\left(e\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + f\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + g\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + h\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$= e\,T^{-1}\left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\right) + f\,T^{-1}\left(\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right) + g\,T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\right) + h\,T^{-1}\left(\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\right) \quad [T^{-1} \text{ is a L.T.}]$$

$$= e(10 - 11x + 3x^2 - x^3) + f(5 - 6x + 2x^2 + x^3) + g(7 - 9x + 3x^2 - x^3) + h(-6 + 7x - 2x^2 + x^3)$$

$$= (10e + 5f + 7g - 6h) + (-11e - 6f + 3g + 7h)\,x + (3e + 2f + 3g - 2h)\,x^2$$
$$+ (-e + f - g + h)\,x^3$$

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<u>Isomorphism</u> $\quad U$ & $V$ are <u>isomorphic</u>.

if <u>there is</u> invertible $T: U \to V$

$T$ is an isomorphism

<u>Ex</u> $\quad M_{22}$ & $P_3$ are isomorphic ($T$ above is an isomorphism)

hard

easy

$$\begin{tikzcd} \underset{\sim}{u}, \underset{\sim}{v} \arrow[r, "T"] \arrow[d, "+"'] & T(\underset{\sim}{u}), T(\underset{\sim}{v}) \arrow[d, "+"] \\ \underset{\sim}{u}+\underset{\sim}{v} \arrow[r, "T"'] & T(\underset{\sim}{u}+\underset{\sim}{v}) \end{tikzcd}$$

$T^{-1}$

$T(\underset{\sim}{u})+T(\underset{\sim}{v}) = T(\underset{\sim}{u}+\underset{\sim}{v})$ (if $T$ is a linear transformation)

$$\underbrace{r(T)}_{\dim(R(T))} + \underbrace{n(T)}_{\dim(K(T))} = \dim(U)$$

domain

$$T: \mathbb{C}^n \to \mathbb{C}^m \quad \boxed{T(\underset{\sim}{x}) = A\underset{\sim}{x}}$$

$m \times n$ $\quad$ $n \times 1$

$$r(A)+n(A)=n$$

$$r(T)+n(T)=\dim(\mathbb{C}^n)=n$$