Math 290 Monday, April 12 Section LT

Tue - Exam E

Thu - ILT

Fri - SLT

BYOB - Food

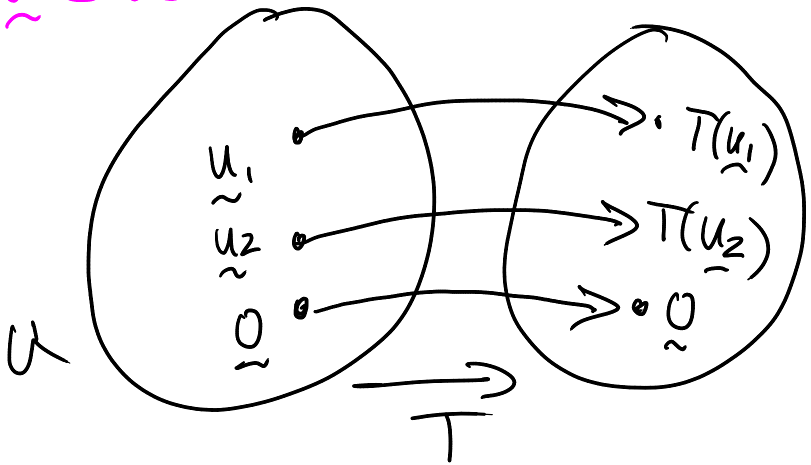
$T: \mathcal{U} \to \mathcal{V}$ function

domain codomain

$T(\underset{\sim}{u}) \in \mathcal{V}$

$\underset{\sim}{u} \in \mathcal{U}$

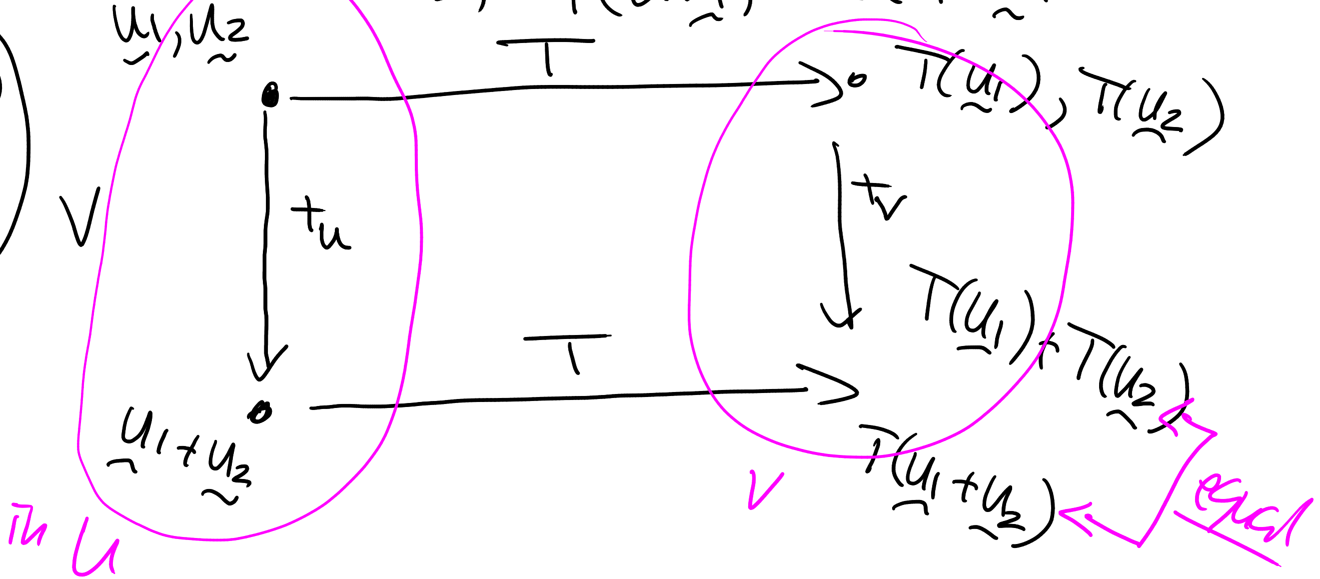
$$\frac{f(x) = x^2}{f: \mathbb{R} \to \mathbb{R}}$$



Defn LT

1) $T(\underset{\sim}{u}_1 + \underset{\sim}{u}_2) = T(\underset{\sim}{u}_1) + T(\underset{\sim}{u}_2)$

2) $T(\alpha \underset{\sim}{u}_1) = \alpha T(\underset{\sim}{u}_1)$



Ex $T : \mathbb{C}^3 \to \mathbb{C}^4$ $\quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -5x_1 + 4x_2 - 6x_3 \\ -6x_1 + 5x_2 - 7x_3 \\ -x_1 + x_2 - x_3 \\ 3x_1 - 2x_2 + 4x_3 \end{bmatrix}$ $\underline{\underline{\text{is}}}$ a L.T.

Could check the definition.

Ex $S : P_3 \to M_{22}$ $\quad S(a + bx + cx^2 + dx^3) = \begin{bmatrix} 2a + b + 3d & a - b + c \\ 5a - 6d & 0 \end{bmatrix}$ is a L.T.

Ex $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} \quad \end{bmatrix} = \begin{bmatrix} -5x_1 \\ -6x_1 \\ -x_1 \\ 3x_1 \end{bmatrix} + \begin{bmatrix} 4x_2 \\ 5x_2 \\ x_2 \\ -2x_2 \end{bmatrix} + \begin{bmatrix} -6x_3 \\ -7x_3 \\ -x_3 \\ 4x_3 \end{bmatrix}$

$$= x_1 \begin{bmatrix} -5 \\ -6 \\ -1 \\ 3 \end{bmatrix} + x_2 \begin{bmatrix} 4 \\ 5 \\ 1 \\ -2 \end{bmatrix} + x_3 \begin{bmatrix} -6 \\ -7 \\ -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 4 & -6 \\ -6 & 5 & -7 \\ -1 & 1 & -1 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \therefore \quad T(\underset{\sim}{x}) = \underline{A}\underset{\sim}{x}$$

Fact $T: U \to V$ $T(\underset{\sim}{0}) = \underset{\sim}{0}$ (Therem LTTZZ)

Proof

$$
\begin{aligned}
\underset{\sim}{0} &= T(\underset{\sim}{0}) - T(\underset{\sim}{0}) \quad (T(\underset{\sim}{0}) + (-T(\underset{\sim}{0}))) \\
&= T(\underset{\sim}{0} + \underset{\sim}{0}) - T(\underset{\sim}{0}) \\
&= T(\underset{\sim}{0}) + \underline{T(\underset{\sim}{0})} - \underline{T(\underset{\sim}{0})} \\
&= T(\underset{\sim}{0}) + \underset{\sim}{0} \\
&= T(\underset{\sim}{0})
\end{aligned}
$$

Ex $S(\underset{\sim}{0}) = S(0 + 0x + 0x^2 + 0x^3)$

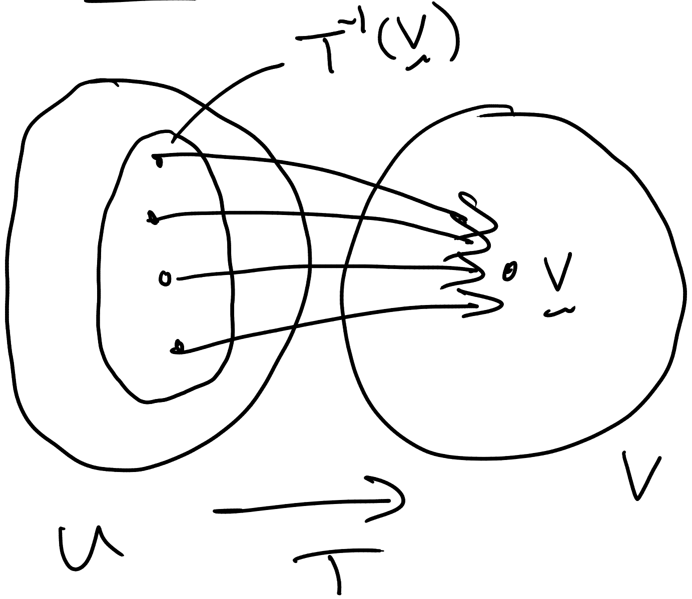
$$
= \begin{bmatrix} 2(0)+0+3(0) & 0-0+0 \\ 5(0)-6(0) & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \underset{\sim}{0}
$$

Pre-Images

$$T(\,?\,) = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 6 \end{bmatrix}$$

pre-image of $\underset{\sim}{v}$

Defn $T: U \to V$, $\underset{\sim}{v} \in V$ $\quad T^{-1}(\underset{\sim}{v}) = \{\underset{\sim}{u} \in U \mid T(\underset{\sim}{u}) = \underset{\sim}{v}\}$



Ex $T^{-1}\left(\begin{bmatrix} -7 \\ -8 \\ -1 \\ 5 \end{bmatrix}\right)$? $\longrightarrow$ $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -7 \\ -8 \\ -1 \\ 5 \end{bmatrix}$

???

$$\begin{bmatrix} -5x_1 + 4x_2 - 6x_3 \\ -6x_1 + 5x_2 - 7x_3 \\ -x_1 + x_2 - x_3 \\ 3x_1 - 2x_2 + 4x_3 \end{bmatrix} = \begin{bmatrix} -7 \\ -8 \\ -1 \\ 5 \end{bmatrix}$$

Solve system w/ augmented matrix

$$\left[\begin{array}{ccc|c} -5 & 4 & -6 & -7 \\ -6 & 5 & -7 & -8 \\ -1 & 1 & -1 & -1 \\ 3 & -2 & 4 & 5 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Infinitely many solutions

$x_1 = 3 - 2x_3$

$x_2 = 2 - x_3$

$$\begin{bmatrix} 3-2x_3 \\ 2-x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$$