Math 290 Monday, April 5 Section SD

Similar Matrices

Defn We say A & B are similar if there exists nonsingular S,

$$S^{-1}AS = B. \leftarrow \text{"similarity transformation"}$$

Also $AS = BS$

similarity is an equivalence relation

Section FS

$$JA = B \leftarrow \text{RREF}$$

matrix multiplication

Diagonalization

similarity

$$S^{-1}AS = D = \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{bmatrix}$$

diagonal matrix

Tue - Exam VS
Laptops

Thu - $\oint$ ME

Fri - Problems
writing E

Mon - LT

Tue - Exam E

Final Exam
Monday, May 10, 8 AM
$\rightarrow$ 11 AM

Theorem DC

A diagonalizable $\iff$ A has a set of n linearly independent eigenvectors.
$n \times n$

Ex $A = \begin{bmatrix} 8 & -6 & 6 \\ 6 & -4 & 6 \\ -3 & 3 & -1 \end{bmatrix}$ (Section E) Eigenvalues: $\lambda = 2$, $\lambda = -1$

$\mathcal{E}_A(2) = \left\langle \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\} \right\rangle$ $\quad \mathcal{E}_A(-1) = \left\langle \left\{ \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix} \right\} \right\rangle$

linearly independent

so basis of $\mathbb{C}^3$

$S = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & -2 \\ -1 & 0 & 1 \end{bmatrix}$ then $S^{-1}AS = \begin{bmatrix} 2 & & 0 \\ & 2 & \\ 0 & & -1 \end{bmatrix}$ $\quad \begin{bmatrix} 2 & & \\ & -1 & \\ & & 2 \end{bmatrix}$

nonsingular

<u>Proof</u> eigenvectors $\Longrightarrow$ diagonalize $\{x_1, x_2, \ldots, x_n\}$ n linearly independent eigenvectors

$$S = [x_1 | x_2 | \ldots | x_n]$$

eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$

$$\begin{aligned} AS &= A[x_1 | x_2 | \ldots | x_n] \\ &= [Ax_1 | Ax_2 | \cdots | Ax_n] = [\lambda_1 x_1 | \lambda_2 x_2 | \ldots | \lambda_n x_n] \\ &= [x_1 | x_2 | \cdots \; x_n] \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} = SD \end{aligned}$$

$$AS = SD \Rightarrow S^{-1}AS = D$$

S non singular

Theorem SUT Every matrix is similar to an upper triangular matrix w/ eigenvalues on the diagonal.