Math 290 Monday, March 22 Section PD

**Defn** V vector space,

$B = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \ldots, \underline{u}_m\}$ then $\dim(B) = m$

The dimension is "well-defined".

Theorem BIS V vector space

B & C bases of V, then B and C have the same size.

**Proof** Suppose $|C| > |B|$ then

C is a linearly dependent set, bigger than a spanning set. Contradiction.

Suppose $|B| > |C|$ then B is a linearly dependent set, bigger than a spanning set. Contradiction.

$\Rightarrow$ So $|B| = |C|$

Tue - Problem Session Writing VS

Thu - EE

Fri - PEE

Chapter E onwards

Runestone Only

Calendar

CP $\rightarrow$ ME

BYOB Music Performers

Facts

$\dim(\mathbb{R}^n) = n$

$\dim(P_n) = n+1$

$\dim(M_{mn}) = m \cdot n$

$\dim(\mathbb{C}) = 2$

Rank, Nullity - important properties of a matrix.

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Section 4.9 (?)

Ex

$$A = \begin{bmatrix} 1 & -1 & -1 & 2 & 0 & -2 \\ 2 & -3 & -5 & 2 & -2 & -14 \\ 1 & -1 & -1 & 2 & 1 & 1 \\ 0 & 2 & 6 & 4 & 1 & 11 \\ 2 & -1 & 1 & 6 & 1 & 3 \end{bmatrix}$$

$$A \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 2 & 4 & 0 & 2 \\ 0 & 1 & 3 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Three non-zero rows

$r = 3$

RANK

$$A^t \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 7 & 5 \\ 0 & 1 & 0 & -2 & -1 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Three non-zero rows

$r = 3$

Theorem G $\dim(V) = t$, $S = \{\underset{\frown}{v_1}, \underset{\frown}{v_2}, \dots, \underset{\frown}{v_m}\}$

1) $m > t$ the set is linearly dependent (SSLD)

2) $m < t$ the set does not span $V$.

3) $m = t$ & $S$ linearly independent $\Rightarrow$ $S$ spans $V$.

4) $m = t$ & $S$ spans $V$ $\Rightarrow$ $S$ linearly independent.

<u>Proof</u> All four parts contradict SSLD

<u>Ex</u> $X = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \ a + 2b - c + 3d = 0 \right\} \subseteq M_{22}$

① $X$ <u>is</u> a subspace of $M_{22}$ (so $X$ is a vector space)

Proofs of closure, TSS

② A basis is $T = \left\{ \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix} \right\}$ (spanning set via $a = -2b + c - 3d$)

automatically linearly independent

(3) Thus $\dim(X) = 3 \quad (= |T|)$

Check this

(4) $R = \left\{ \begin{bmatrix} -7 & -2 \\ 1 & 4 \end{bmatrix}, \begin{bmatrix} -18 & 3 \\ 3 & 5 \end{bmatrix}, \begin{bmatrix} -19 & 4 \\ 1 & 4 \end{bmatrix} \right\} \subseteq X$

Check that $R$ spans $X$. Hard. Solve $\alpha_1 \begin{bmatrix} -7 & -2 \\ 1 & 4 \end{bmatrix} + \alpha_2 \begin{bmatrix} -18 & 3 \\ 3 & 5 \end{bmatrix} + \alpha_3 \begin{bmatrix} -19 & 4 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$?

in $X$

Yes. Since $|R| = 3 = \dim(X)$ + spanning

$\Rightarrow R$ linearly independent.

(5) Start over, $R$ is linearly independent (EZ) homogeneous system

$|R| = 3 = \dim X$ + lin. ind. $\Rightarrow R$ spanning set for $X$

(6) $P = \left\{ \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} -1 & -5 \\ -2 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 4 \\ 2 & -3 \end{bmatrix} \right\} \subseteq X$

too big $\Rightarrow$ linearly dependent set.