Math 290 Friday, March 19 Section D

Mon - PD
Tue - Problems writing VS
on Runestone only
Thu - EE
Fri - PEE

Defn Let V be vector space, and $B = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \ldots, \underset{\sim}{u}_n\}$ is a basis of V. Then the dimension of V is n. write $\dim(V) = n$.

[size of B is n.]

Theorem SSLD $S = \{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \ldots, \underset{\sim}{v}_t\}$ span V. Then any set of $t+1$ or more vectors is linearly dependent.

Proof Grab $R = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \ldots, \underset{\sim}{u}_m\}$ arbitrary vectors, $m > t$

Can write each $\underset{\sim}{u}_j$ as a linear combo of the $\underset{\sim}{v}_i$.

$$\underset{\sim}{u}_1 = a_{11}\underset{\sim}{v}_1 + a_{21}\underset{\sim}{v}_2 + \cdots\cdots + a_{t1}\underset{\sim}{v}_t$$

$$\underset{\sim}{u}_2 = a_{12}\underset{\sim}{v}_1 + a_{22}\underset{\sim}{v}_2 + \cdots\cdots + a_{t2}\underset{\sim}{v}_t$$

$$\vdots$$

$$\underset{\sim}{u}_m = a_{1m}\underset{\sim}{v}_1 + a_{2m}\underset{\sim}{v}_2 + \cdots\cdots + a_{tm}\underset{\sim}{v}_t$$

$$\underset{\sim}{u}_j = \sum_{i=1}^{t} a_{ij}\underset{\sim}{v}_i \qquad 1 \le j \le m$$

Get a_{ij} $1 \le i \le t$, $1 \le j \le m$

Build a system of equations, t equations, m variables $(x_1, x_2, \ldots, x_m)$ homogenous.

$$\boxed{a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1m}x_m} = 0$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2m}x_m = 0$$

$$\vdots$$

$$a_{t1}x_1 + a_{t2}x_2 + a_{t3}x_3 + \cdots + a_{tm}x_m = 0$$

$$\sum_{j=1}^{m} a_{ij}x_j = 0 \qquad 1 \le i \le t$$

m variable, t equations, $m > t$ HMVEI $\Rightarrow$ infinitely many solutions

Choose one non-zero/non-trivial/non-Grant solution

$$x_1 = c_1, \; x_2 = c_2, \; \dots, \; x_m = c_m$$

Form

$$c_1 u_1 + c_2 u_2 + \dots + c_m u_m$$

$$= c_1(a_{11} v_1 + a_{21} v_2 + \dots + a_{t1} v_t)$$

$$+ c_2(a_{12} v_1 + a_{22} v_2 + \dots + a_{t2} v_t)$$

$$\vdots$$

$$+ c_m(a_{1m} v_1 + a_{2m} v_2 + \dots + a_{tm} v_t)$$

$$= (c_1 a_{11} + c_2 a_{12} + \dots + c_m a_{1m}) v_1$$

$$+ (c_1 a_{21} + c_2 a_{22} + \dots + c_m a_{2m}) v_2$$

$$\vdots$$

$$+ (c_1 a_{t1} + c_2 a_{t2} + \dots + c_m a_{tm}) v_t$$

$$= 0 v_1 + 0 v_2 + \dots + 0 v_t$$

c_j's are solution to the homogeneous system

$$x_i = c_i \qquad 1 \le i \le m$$

$$\sum_{j=1}^{m} c_j u_j$$

$$= \sum_{j=1}^{m} c_j \left(\sum_{i=1}^{t} a_{ij} v_i \right)$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{t} c_j a_{ij} v_i$$

$$= \sum_{i=1}^{t} \sum_{j=1}^{m} c_j a_{ij} v_i$$

$$= \sum_{i=1}^{t} \left(\sum_{j=1}^{m} c_j a_{ij} \right) v_i$$

$$= \sum_{i=1}^{t} 0 v_i$$

$= \underset{\sim}{0} + \underset{\sim}{0} + \dots + \underset{\sim}{0} = \underset{\sim}{0}$

So R <u>is</u> linearly dependent by this non-trivial RLD.