Math 290 Thursday, March 18 Section B

Fri – D (proof)
BYOB – Pets

Mon – PD

Thu – Problem Session
writing VS

<u>Defn</u> Given a vector space V, a linearly independent spanning set $B = \{\underline{u}_1, \underline{u}_2, \dots, \underline{u}_n\}$ is a <u>basis</u> of V.

Theorems BNS, BCS, BS, BRS

"Standard" bases

Defn
<u>SUV</u>

$\mathbb{C}^n$ $\left\{ \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \dots \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \right\}$ Columns of I_n

P_n $\{1, x, x^2, x^3, \dots, x^n\}$

M_{mn} $\left\{ \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ & & \vdots & & \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \dots \begin{bmatrix} 0 & 0 & \cdots & 0 \\ & \vdots & & \\ 0 & 0 & 0 & 1 \end{bmatrix} \right\}$ total of mn matrices

Another basis of P_n : $\{1, 1+x, 1+x+x^2, \ldots, 1+x+x^2+\cdots+x^n\}$

Any vector space will likely have infinitely many different bases.

Ex $W = \left\langle \left\{ \begin{bmatrix} -4 \\ -2 \\ 3 \\ -11 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} \right\} \right\rangle \leq \mathbb{C}^4$

Basis of W? A = matrix w/ vectors as columns, $W = R(A^t)$ (use BRS)

$A^t \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ BRS says take non zero rows

$B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\}$

Grab $\begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix} \in W$ then $\begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix} = (-2)\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + 7\begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + 3\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}$

uniquely

Theorem VRRB

Ex $U = \{a+bx+cx^2+dx^3 \mid a+2b+5d=0,\ a-b+3c-2d=0\} \leq P_3$

- Is a subspace of P_3

Find a basis of U

$$a+2b+5d=0$$
$$a-b+3c-2d=0$$

coefficient matrix of homogeneous system

$$\begin{bmatrix} 1 & 2 & 0 & 5 \\ 1 & -1 & 3 & -2 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 3 \end{bmatrix}$$

dependent variables

$$a = -2c-d$$
$$b = c-3d$$

$$U = \{(-2c-d)+(c-3d)x+\underbrace{cx^2+dx^3}\mid \underbrace{c,d\in\mathbb{C}}\}$$

c, d are free

$$=\{(-2c+cx+cx^2)+(-d-3dx+dx^3)\mid c,d\in\mathbb{C}\}$$

$$=\{c(\underline{-2+x+x^2})+d(\underline{-1-3x+x^3})\mid c,d\in\mathbb{C}\}=\langle\{-2+x+x^2,\ -1-3x+x^3\}\rangle$$

$T=\{-2+x+x^2, -1-3x+x^3\}$ spans U. Bonus: T is linearly independent, needs check.

$$\alpha(-2+x+x^2)+\beta(1-3x+x^3)=\underset{\sim}{0}=0+0x+0x^2+0x^3$$

$$(-2\alpha+\beta)+(\alpha-3\beta)x+\alpha x^2+\beta x^3=0+0x+0x^2+0x^3 \implies \alpha=0,\ \beta=0$$

Theorem COB $\quad B=\{\underset{\sim}{v}_1,\underset{\sim}{v}_2,\ldots,\underset{\sim}{v}_p\}$ orthonormal <u>basis</u> of $\mathbb{C}^n$

$$\underset{\sim}{w}=\langle \underset{\sim}{v}_1,\underset{\sim}{w}\rangle \underset{\sim}{v}_1+\langle \underset{\sim}{v}_2,\underset{\sim}{w}\rangle \underset{\sim}{v}_2+\cdots+\langle \underset{\sim}{v}_p,\underset{\sim}{w}\rangle \underset{\sim}{v}_p$$

<u>Ex</u> $\left\{\begin{bmatrix}1/3\\2/3\\2/3\end{bmatrix},\begin{bmatrix}2/3\\-2/3\\1/3\end{bmatrix},\begin{bmatrix}2/3\\1/3\\-2/3\end{bmatrix}\right\}$ orthonormal basis of $\mathbb{C}^3$

$$\langle \underset{\sim}{w},\underset{\sim}{v}_1\rangle = 3(1/3)+(-3)(2/3)+6(2/3)$$
$$=1-2+4=3$$

$$\underset{\sim}{w}=\begin{bmatrix}3\\-3\\6\end{bmatrix}=3\begin{bmatrix}1/3\\2/3\\2/3\end{bmatrix}+6\begin{bmatrix}2/3\\-2/3\\1/3\end{bmatrix}+(-3)\begin{bmatrix}2/3\\1/3\\-2/3\end{bmatrix}$$