Math 290 Monday, March 15 Section LISS

Tue - Problem Session

Thu - B

Fri - D

BYOB: Pets

Mon - PD

V - vector space

$+$ - vector addition

- scalar multiplication

$\Rightarrow$ linear combination:

$\alpha_1 \underset{\sim}{v_1} + \alpha_2 \underset{\sim}{v_2} + \cdots + \alpha_n \underset{\sim}{v_n}$

span: <u>all</u> linear combinations

RLD: $\alpha \underset{\sim}{v_1} + \cdots + \alpha_n \underline{v_n} = \underset{\sim}{0}$ ← in <u>every</u> vector space

linear independence

Ex In M_{22}, $T = \left\{ \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \right\}$ is linearly independent.

RLD: $\alpha_1 \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} + \alpha_2 \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} + \alpha_3 \begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} = \underset{\sim}{0}$

$$\begin{bmatrix} 2\alpha_1 + \alpha_2 + 2\alpha_3 & \alpha_1 + \alpha_3 \\ -\alpha_1 + 3\alpha_2 + \alpha_3 & 2\alpha_1 + \alpha_2 - \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Means

$$\begin{aligned} 2\alpha_1 + \alpha_2 + 2\alpha_3 &= 0 \\ \alpha_1 + \alpha_3 &= 0 \\ -\alpha_1 + 3\alpha_2 + \alpha_3 &= 0 \\ 2\alpha_1 + \alpha_2 - \alpha_3 &= 0 \end{aligned}$$

$$\underset{\text{coefficient matrix}}{\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 1 \\ -1 & 3 & 1 \\ 2 & 1 & -1 \end{bmatrix}} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

the $\boxed{\text{only}}$ solution is $\alpha_1 = \alpha_2 = \alpha_3 = 0$.

So T is linearly independent.

Ex Show that $R = \{1-x+2x^2, 1+3x^2, 1-x+3x^2\}$ spans P_2.

Is R a spanning set for/of P_2? Is $\langle R \rangle = P_2$?

$\langle R \rangle \subseteq P_2$. EZ $\qquad P_2 \subseteq \langle R \rangle$

$$a+bx+cx^2 = a_1(1-x+2x^2) + a_2(1+3x^2) + a_3(1-x+3x^2)$$

Do a_1, a_2, a_3 exist?

$$(a_1+a_2+a_3) + (-a_1-a_3)x + (2a_1+3a_2+3a_3)x^2 = a+bx+cx^2$$

Equality of polynomials $\Rightarrow$

$$\begin{aligned} a_1+a_2+a_3 &= a \\ -a_1 \quad\; -a_3 &= b \\ 2a_1+3a_2+3a_3 &= c \end{aligned} \qquad \left[\begin{array}{ccc|c} 1 & 1 & 1 & a \\ -1 & 0 & -1 & b \\ 2 & 3 & 3 & c \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3a-c \\ 0 & 1 & 0 & a+b \\ 0 & 0 & 1 & -3a-b+c \end{array}\right]$$

This system has a (unique) solution for every choice of $a, b, \& c$. So every polynomial in P_2 <u>is</u> a linear combination of the vectors in R. So R spans P_2.

$\underline{Ex}$ Find a spanning set for

$$W = \{ a+bx+cx^2 \mid a-4b+3c=0 \} \leq P_2 \quad \text{vector space}$$

$\underline{\text{subspace}}$ (vector space)

$a-4b+3c=0 \qquad N([1\ -4\ 3]) \qquad a = 4b-3c$

$\hookrightarrow$ RREF $[1\ -4\ 3] \qquad b,c \in \mathbb{C}$

$$W = \{ (4b-3c)+bx+cx^2 \mid b,c \in \mathbb{C} \}$$

$$= \{ (4b+bx)+(-3c+cx^2) \mid b,c \in \mathbb{C} \}$$

$$= \{ b(4+x)+c(-3+x^2) \mid b,c \in \mathbb{C} \}$$

$$= \langle \underline{\{ 4+x, -3+x^2 \}} \rangle$$

a spanning set ($\underline{\text{also}}$ linearly independent)