Math 290 Friday, March 12 Section 5

Mon- LISS

Tue- Problems

Thu- B

Fri- D

Ex $S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \middle| \, 5x_1 - x_2 + 8x_3 = 0 \right\} \subseteq \mathbb{C}^3$

Could check 10 properties ($\underset{\sim}{u} + \underset{\sim}{v} = \underset{\sim}{v} + \underset{\sim}{u}$ ✓)

Apply Theorem TSS

① S non-empty? $\begin{bmatrix} 0 \\ 8 \\ 1 \end{bmatrix} \in S$. Yes.

$S = N([5 \ -1 \ 8])$

Better: is $\underset{\sim}{0} \in S$? $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \in S$. Yes

② Suppose $\underset{\sim}{x} \in S$ & $\underset{\sim}{y} \in S$.

$\underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ then $5x_1 - x_2 + 8x_3 = 0$

$\underset{\sim}{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ then $5y_1 - y_2 + 8y_3 = 0$

$\underset{\sim}{x} + \underset{\sim}{y} = \begin{bmatrix} x_1 + y_1 \\ x_2 + y_2 \\ x_3 + y_3 \end{bmatrix}$. (Is $\underset{\sim}{x} + \underset{\sim}{y} \in S$?)

Look at (consider, examine),

$5(x_1 + y_1) - (x_2 + y_2) + 8(x_3 + y_3)$

$= 5x_1 + 5y_1 - x_2 - y_2 + 8x_3 + 8y_3$

$= (5x_1 - x_2 + 8x_3) + (5y_1 - y_2 + 8y_3)$

$= 0 + 0 = 0$. So $\underset{\sim}{x} + \underset{\sim}{y} \in S$

③ Suppose $\alpha \in \mathbb{C}$, $\underset{\sim}{x} \in S$ $\underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, know $5x_1 - x_2 + 8x_3 = 0$

$\alpha \underset{\sim}{x} = \begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{bmatrix}$. look at $5(\alpha x_1) - (\alpha x_2) + 8(\alpha x_3)$

$= \alpha(5x_1 - x_2 + 8x_3) = 5 \cdot 0 = 0$

So $\alpha x \in S$.

By Theorem TSS, S is a subspace of $\mathbb{C}^3$, (and hence a vector space in its own right)

Ex $R = \langle \{1 - 2x + x^2 + 3x^3, 2 + 3x^2 + x^3\} \rangle \subseteq P_3 \leftarrow$

(R is a subspace, Theorem SSS)

$1 + 2x + 2x^2 - 2x^3 \in R$?

Is $1 + 2x + 2x^2 - 2x^3 = a(1 - 2x + x^2 + 3x^3) + b(2 + 3x^2 + x^3)$?

$= (a - 2ax + ax^2 + 3ax^3) + (2b + 3bx^2 + bx^3)$

$= (a + 2b) + (-2a)x + (a + 3b)x^2 + (3a + b)x^3$

Equate coefficients

$$a+2b=1$$
$$-2a=2$$
$$a+3b=2$$
$$3a+b=-2$$

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ -2 & 0 & 2 \\ 1 & 3 & 2 \\ 3 & 1 & -2 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

augmented matrix of the system

Consistent system $(a=-1, b=1)$, so there are scalars

So yes $1+2x+2x^2-2x^3 \in R$

Theorem $N(A)$ is a subspace.

Proof ① $N(A)$ non-empty?

$A\underset{\sim}{0} = \underset{\sim}{0} \Rightarrow \underset{\sim}{0} \in N(A)$.

② Suppose $\underset{\sim}{x}, \underset{\sim}{y} \in N(A)$. Know $A\underset{\sim}{x} = \underset{\sim}{0}$, $A\underset{\sim}{y} = \underset{\sim}{0}$

Look at $A(\underset{\sim}{x}+\underset{\sim}{y}) = A\underset{\sim}{x} + A\underset{\sim}{y} = \underset{\sim}{0} + \underset{\sim}{0} = \underset{\sim}{0}$. So $\underset{\sim}{x}+\underset{\sim}{y} \in N(A)$

(3) Suppose $\underset{\sim}{x} \in N(A)$, $\alpha \in \mathbb{C}$. Know $A\underset{\sim}{x} = \underset{\sim}{0}$

Then $A(\alpha \underset{\sim}{x}) = \alpha(A\underset{\sim}{x}) = \alpha \underset{\sim}{0} = \underset{\sim}{0}$. So $\alpha \underset{\sim}{x} \in N(A)$

By Theorem TSS, $N(A)$ subspace of $\mathbb{C}^n$.