Math 290 Monday, March 1 Section MINM

Theory Day

Tuesday - Problems
Thu - CRS
Fri - FS
Sage x2

BYOB - solo outdoor sports

Mon - Problems
Writing M

Tue - Exam M

Theorem NPNF : AB nonsingular
$\iff$
A & B both nonsingular

2nd Grade :

$xy = 0 \iff x = 0$ or $y = 0$

$xy \neq 0 \iff x \neq 0$ and $y \neq 0$

non zero $\longleftrightarrow$ non singular

zero $\cong$ singular

"singular is to matrices as zero is to numbers"

Proof ($\Rightarrow$) AB nonsingular $\Rightarrow$ A nonsingular & B nonsingular

Contrapositive: A singular or B singular $\Rightarrow$ AB singular

Case 1 B singular so there is $\underset{\sim}{z} \neq \underset{\sim}{0}$ so that $B\underset{\sim}{z} = \underset{\sim}{0}$.

Then $(AB)\underset{\sim}{z} = A(B\underset{\sim}{z}) = A\underset{\sim}{0} = \underset{\sim}{0}$, so AB is singular

Case 2 B nonsingular. Then A is singular. (hypothesis)

There exists $\underset{\sim}{y} \neq \underset{\sim}{0}$ so that $A\underset{\sim}{y} = \underset{\sim}{0}$. $B\underset{\sim}{w} = \underset{\sim}{y}$.

Solve $LS(B, \underset{\sim}{y})$. Unique solution (NMUS), call it $\underset{\sim}{w}$

Claim $\underset{\sim}{w} \neq \underset{\sim}{0}$. Suppose $\underset{\sim}{w} = \underset{\sim}{0}$. Then $\underset{\sim}{y} = B\underset{\sim}{w} = B\underset{\sim}{0} = \underset{\sim}{0}$. $\cancel{\Rightarrow}$

Now $(AB)\underset{\sim}{w} = A(B\underset{\sim}{w}) = A\underset{\sim}{y} = \underset{\sim}{0}$. So AB is singular

$(\Leftarrow)$ A & B nonsingular $\Rightarrow$ AB nonsingular

Consider $\mathcal{LS}(AB, \underset{\sim}{0})$ Solve

$(AB)\underset{\sim}{x} = \underset{\sim}{0}$

$A(B\underset{\sim}{x}) = \underset{\sim}{0}$ $\xRightarrow{A \text{ nonsingular}}$ $B\underset{\sim}{x} = \underset{\sim}{0}$ $\xRightarrow{B \text{ nonsingular}}$ $\underset{\sim}{x} = \underset{\sim}{0}$

So AB is nonsingular.

Theorem OSIS $AB = I \Rightarrow BA = I$

Proof $\underset{\text{nonsingular NMRRI}}{I} = AB$ $\xRightarrow{\text{NPNF}}$ B nonsingular (A nonsingular)

Theorem CINM $\Rightarrow C$ so that $BC = I$

Then $BA = (BA)I = (BA)(BC)$
$= B(AB)C = BIC = BC = I$

Theorem NI   A nonsingular $\iff$ A invertible

$(\Leftarrow)$ A invertible, A^{-1} exists $I_n = AA^{-1}$ $\Longrightarrow_{\text{NPNF}}$ A nonsingular

$\uparrow$ nonsingular NMRRI

$(\Rightarrow)$ A nonsingular $\xrightarrow[\text{CINM gives B}]{}$ $AB = I$ $\xrightarrow[\text{OSIS}]{}$ $BA = I \Rightarrow B = A^{-1}$

Unitary Matrices

$U^{-1} = U^*$

Theorem UMPIP

$\langle U\tilde{u}, U\tilde{v}\rangle = \langle \tilde{u}, \tilde{v}\rangle$

$\tilde{u} \cdot \tilde{v} = \|\tilde{u}\| \|\tilde{v}\| \cos\theta$ $\|U\tilde{v}\| = \|\tilde{v}\|$