Math 290 Friday, February 26 Section MISLE

Mon- MINM
Tue- Problems
Thu- CRS
Fri- FS
Sage
Mon- Problems
Writing

Ex

$$\begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 6 & 11 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 14 & \\ & \end{bmatrix}$$

Most of the time, $AB \neq BA$

Ex $\mathbb{R}$ $xy = 0 \Rightarrow x=0, y=0$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Matrices $AB = \mathcal{O} \not\Rightarrow A = \mathcal{O}$ or $B = \mathcal{O}$

$$\begin{bmatrix}1&2\\2&4\end{bmatrix}\begin{bmatrix}-3&-2\\1&5\end{bmatrix}=\begin{bmatrix}-1&8\\-2&16\end{bmatrix}$$

$$\begin{bmatrix}1&2\\2&4\end{bmatrix}\begin{bmatrix}1&2\\2&3\end{bmatrix}$$

$\dfrac{\text{MISLE}}{\mathbb{R}}$

$2x=8 \rightarrow \frac{1}{2}(2x)=\frac{1}{2}8 \rightarrow (\frac{1}{2}2)x=\frac{8}{2}$

$\rightarrow 1x=4$

$\rightarrow x=4$

unique solution

$\frac{1}{2}$ is the multiplicative inverse

$0x=8$ no solution

$0x=0$ infinitely many solutions

Ex

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & 4 & -1 \\ -1 & -1 & 0 & 1 \\ 2 & 3 & 5 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 10 & 5 & 7 & -6 \\ -11 & -6 & -9 & 7 \\ 3 & 2 & 3 & -2 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I_4 \qquad B = A^{-1}$$

SLEMM-ification

Ex Solve $A\underset{\sim}{x} = \underset{\sim}{b}$, $\underset{\sim}{b} = \begin{bmatrix} 7 \\ 6 \\ -2 \\ 14 \end{bmatrix}$

$$A^{-1}(A\underset{\sim}{x}) = A^{-1}\underset{\sim}{b}$$

$$(A^{-1}A)\underset{\sim}{x} = A^{-1}\underset{\sim}{b}$$

$$I_4\underset{\sim}{x} = A^{-1}\underset{\sim}{b}$$

$$\underset{\sim}{x} = A^{-1}\underset{\sim}{b}$$

$$\underset{\sim}{x} = A^{-1}b = \begin{bmatrix} 10 & 5 & 7 & -6 \\ -11 & -6 & -9 & 7 \\ 3 & 2 & 3 & -2 \\ -1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ 6 \\ -2 \\ 14 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \\ 3 \end{bmatrix}$$

Ex Inverse of $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 2 \\ 2 & 3 & 6 \end{bmatrix}$?

Want B so that $AB = I_3$; $A[B_1 | B_2 | B_3] = [e_1 | e_2 | e_3]$

$AB_1 = e_1$ $\quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -1 & 0 & 2 & 0 \\ 2 & 3 & 6 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & -3 \end{array}\right]$ $\leftarrow B_1$

$AB_2 = e_2$ $\quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -1 & 0 & 2 & 1 \\ 2 & 3 & 6 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -1 \end{array}\right]$ $\leftarrow B_2$

$AB_3 = e_3$ $\quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -1 & 0 & 2 & 0 \\ 2 & 3 & 6 & 1 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{array}\right]$ $\leftarrow B_3$

$B = \begin{bmatrix} -6 & -3 & 2 \\ 10 & 4 & -3 \\ -3 & -1 & 1 \end{bmatrix}$ $\quad AB = I_3$ & $BA = I_3$

CINM $\quad A$ nonsingular

$[A | I_n] \xrightarrow[\text{RREF}]{} [I_n | A^{-1}]$

Theorem (Sox & Shoes) $(AB)^{-1} = B^{-1}A^{-1}$

Proof AB $(AB)^{-1}$? $B^{-1}A^{-1}$

$$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$$

$$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = B^{-1}B = I$$

So the inverse of AB <u>is</u> $B^{-1}A^{-1}$

$$B^{-1}A^{-1} = (AB)^{-1}$$