Math 290 Monday, February 15 Section ①

Ex $\underset{\sim}{u} = \begin{bmatrix} 2+i \\ 4-3i \end{bmatrix}$, $\underset{\sim}{v} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$ in $\mathbb{C}^2$

$$\langle \underset{\sim}{u}, \underset{\sim}{v} \rangle = \overline{2+i}\,(5) + \overline{4-3i}\,(2)$$
$$= (2-i)(5) + (4+3i)(2)$$
$$= 10-5i + 8+6i$$
$$= 18+i \leftarrow \text{SCALAR}$$

SCALAR

span

$\langle S \rangle$

$\langle \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \underset{\sim}{u}_3\} \rangle$

Tue – Problem Session Writing V

Mon – MO$^+$

Tue – Exam V

Thu – MM

Fri – MSLE

Theorem CRVA $\overline{\underset{\sim}{x}+\underset{\sim}{y}} = \overline{\underset{\sim}{x}} + \overline{\underset{\sim}{y}}$

Defn CCCV. Define $\overline{\underset{\sim}{x}}$.

$$[\overline{\underset{\sim}{x}}]_i = \overline{[\underset{\sim}{x}]_i}$$

Proof For $1 \le i \le n$

$$[\overline{\underset{\sim}{x}+\underset{\sim}{y}}]_i = \overline{[\underset{\sim}{x}+\underset{\sim}{y}]_i} = \overline{[\underset{\sim}{x}]_i + [\underset{\sim}{y}]_i}$$
$$= \overline{[\underset{\sim}{x}]_i} + \overline{[\underset{\sim}{y}]_i} = [\overline{\underset{\sim}{x}}]_i + [\overline{\underset{\sim}{y}}]_i = [\overline{\underset{\sim}{x}}+\overline{\underset{\sim}{y}}]_i.$$

So by Defn CVE $\overline{\underset{\sim}{x}+\underset{\sim}{y}} = \overline{\underset{\sim}{x}} + \overline{\underset{\sim}{y}}$

IPVA $\langle \underset{\sim}{u}+\underset{\sim}{v}, \underset{\sim}{w} \rangle = \langle \underset{\sim}{u}, \underset{\sim}{w} \rangle + \langle \underset{\sim}{v}, \underset{\sim}{w} \rangle$

IPSM $\langle \alpha \underset{\sim}{u}, \underset{\sim}{v} \rangle = \bar{\alpha} \langle \underset{\sim}{u}, \underset{\sim}{v} \rangle$

$\langle \underset{\sim}{u}, \alpha \underset{\sim}{v} \rangle = \alpha \langle \underset{\sim}{u}, \underset{\sim}{v} \rangle$

IPAC $\langle \underset{\sim}{u}, \underset{\sim}{v} \rangle = \overline{\langle \underset{\sim}{v}, \underset{\sim}{u} \rangle}$

PIP $\langle \underset{\sim}{u}, \underset{\sim}{u} \rangle \geq 0$ w/ equality if and only if $\underset{\sim}{u} = \underset{\sim}{0}$.

Defn $\underset{\sim}{u}, \underset{\sim}{v}$ orthogonal if $\langle \underset{\sim}{u}, \underset{\sim}{v} \rangle = 0$ "perpendicular"

Defn $\{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \dots, \underset{\sim}{v}_n\}$ orthogonal if $\langle \underset{\sim}{v}_i, \underset{\sim}{v}_j \rangle = 0$ when $i \neq j$.

Gram-Schmidt $\qquad 2\vec{i}+3\vec{j}-6\vec{k}$

set $\longrightarrow$ orthogonal set w/ the same span.

Theorem OSLI

S is an orthogonal set of nonzero vectors. $\Rightarrow$ S linearly independent.

Proof Start w/ a RLD $a_1\underset{\sim}{u}_1+a_2\underset{\sim}{u}_2+\cdots+a_n\underset{\sim}{u}_n=\underset{\sim}{0}$

$S=\{\underset{\sim}{u}_1,\underset{\sim}{u}_2,\ldots,\underset{\sim}{u}_n\}$

① $\langle \underset{\sim}{u}_i, a_1\underset{\sim}{u}_1+a_2\underset{\sim}{u}_2+\cdots+a_n\underset{\sim}{u}_n\rangle=\langle \underset{\sim}{u}_i,\underset{\sim}{0}\rangle=0$

② $\langle \underset{\sim}{u}_i, a_1\underset{\sim}{u}_1+a_2\underset{\sim}{u}_2+\cdots+a_n\underset{\sim}{u}_n\rangle=\langle \underset{\sim}{u}_i,a_1\underset{\sim}{u}_1\rangle+\langle \underset{\sim}{u}_i,a_2\underset{\sim}{u}_2\rangle+\cdots+\langle \underset{\sim}{u}_i,a_n\underset{\sim}{u}_n\rangle$

$=a_1\langle \underset{\sim}{u}_i,\underset{\sim}{u}_1\rangle+a_2\langle \underset{\sim}{u}_i,\underset{\sim}{u}_2\rangle+\cdots+a_n\langle \underset{\sim}{u}_i,\underset{\sim}{u}_n\rangle$

$=a_1\cdot 0+a_2\cdot 0+\cdots+a_i\langle \underset{\sim}{u}_i,\underset{\sim}{u}_i\rangle+\cdots+a_n 0$

$=a_i\langle \underset{\sim}{u}_i,\underset{\sim}{u}_i\rangle$

nonzero, PIP

Now ①② $\Rightarrow$ $0=a_i\langle \underset{\sim}{u}_i,\underset{\sim}{u}_i\rangle \Rightarrow a_i=0$ for all i!

2nd circle

The RLD is only true w/ all the scalars zero. S is linearly independent.