Math 290 Thursday, Feb 11

Section LI

Fri - LDS

Mon - O

Tue - Problem Session Writing V

BREAK

Defn $S = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_k}\}$ is <u>linearly independent</u> if whenever

$$a_1 \underset{\sim}{v_1} + a_2 \underset{\sim}{v_2} + \cdots + a_k \underset{\sim}{v_k} = \underset{\sim}{0}$$

← "Relation of Linear Dependence" RLD

then $a_1 = a_2 = \cdots = a_k = 0$

<u>Ex</u> $S = \left\{ \begin{bmatrix} 4 \\ 3 \\ 4 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 4 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 10 \\ -7 \\ -4 \\ 9 \\ -1 \end{bmatrix} \right\} \subseteq \mathbb{C}^5$ linearly independent?

$= \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \underset{\sim}{v_3}\}$

Notice:

$$-1 \begin{bmatrix} 4 \\ 3 \\ 4 \\ 5 \\ 7 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ 5 \\ 4 \\ -2 \\ 4 \end{bmatrix} + 1 \begin{bmatrix} 10 \\ -7 \\ -4 \\ 9 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

This set is <u>not</u> linearly independent.

Scalars $a_1 \underset{\sim}{v_1} + a_2 \underset{\sim}{v_3} + a_3 \underset{\sim}{v_3} = \underset{\sim}{0}$? (RLD) (non-trivial scalars ?)

SLSLC $\Rightarrow$ system (homogenous)

$$\left[\begin{array}{ccc|c} 4 & -3 & 10 & 0 \\ 3 & 5 & -7 & 0 \\ 4 & 4 & -4 & 0 \\ 5 & -2 & 9 & 0 \\ 7 & 4 & -1 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

there <u>are</u> non-trivial solutions

($2 = r < n = 3$)

(For example $x_1 = -1$, $x_2 = 2$, $x_3 = 1$)

<u>OR</u> Appeal to Theorem LIVRN

matrix w/ S as columns

$$\begin{bmatrix} 4 & 3 & 10 \\ 3 & 5 & -7 \\ 4 & 4 & -4 \\ 5 & -2 & 9 \\ 7 & 4 & -1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$r = 2$, $n = 3$

$\underline{2 < 3} \Rightarrow$ linearly dependent

Ex $S = \left\{ \begin{bmatrix} 0 \\ 8 \\ -3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ 8 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 8 \\ 3 \\ -2 \\ 3 \\ -2 \end{bmatrix} \right\} \subseteq \mathbb{C}^5$

$= \{ \underset{\sim}{x}_1, \underset{\sim}{x}_2, \underset{\sim}{x}_3 \}$

LIVRN

$[\underset{\sim}{x}_1 | \underset{\sim}{x}_2 | \underset{\sim}{x}_3] \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$r = 3 = n$

$\Rightarrow$ linearly independent set

Theorem NMLIC

A nonsingular $\iff$ columns of A are linearly independent

Theorem MVSLD

$\left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ -6 \\ 4 \end{bmatrix}, \begin{bmatrix} 9 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix} \right\}$ 4 vectors from $\mathbb{C}^3$

$4 > 3 \Rightarrow$ linearly dependent

Ex $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 7 \\ 9 \end{bmatrix} \right\}$ add

linearly dependent set $1\underset{\sim}{v}_1 + 1\underset{\sim}{v}_2 - \underset{\sim}{v}_3 = \underset{\sim}{0}$

/ System w/ 5 equations, 7 variables, →

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 6 & 0 & -7 & 5 & 0 & -3 \\ 0 & 1 & 3 & 0 & 9 & -4 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 8 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_1, x_2, x_3, x_7 dependent

$D = \{1, 2, 4, 7\}$

① Null space of coefficient matrix (last column now zeros)

$F = \{3, 5, 6, 8\}$

not a pivot $\Rightarrow$ consistent RCLS

x_3 x_5 x_6 free

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -3 - 6x_3 + 7x_5 - 5x_6 \\ 2 - 3x_3 - 9x_5 + 4x_6 \\ x_3 \\ 1 - 0x_3 - 2x_5 - 8x_6 \\ x_5 \\ x_6 \\ 3 - 0x_3 - 0x_5 - 0x_6 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -6 \\ -3 \\ \boxed{1} \\ -0 \\ \boxed{0} \\ \boxed{0} \\ -0 \end{bmatrix} + x_5 \begin{bmatrix} 7 \\ -9 \\ \boxed{0} \\ -2 \\ \boxed{1} \\ \boxed{0} \\ -0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 4 \\ \boxed{0} \\ -8 \\ \boxed{0} \\ \boxed{1} \\ -0 \end{bmatrix}$$

D = "nice pattern of zeros & ones"

VFSLS

$$N(A) = \left\langle \left\{ \begin{bmatrix} -6 \\ -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -9 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 0 \\ -8 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \right\rangle$$

First, Second, Third, Fourth

2 Original system (not null space)

Solution: $x_3 = x_5 = x_6 = 0$

-3

Theorem $\boxed{PS}\boxed{PHS}$

Ex

$$a_1 \begin{bmatrix} x \\ x \\ 1 \\ x \\ 0 \\ 0 \\ x \end{bmatrix} + a_2 \begin{bmatrix} x \\ x \\ 0 \\ x \\ 1 \\ 0 \\ x \end{bmatrix} + a_3 \begin{bmatrix} x \\ x \\ 0 \\ x \\ 0 \\ 1 \\ x \end{bmatrix} = \begin{bmatrix} x \\ x \\ a_1 \\ x \\ a_2 \\ a_3 \\ x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

"pattern of zeros and ones"

we conclude that $a_1 = a_2 = a_3 = 0$

So the set is linearly independent.

Theorem BNS ~~SSNS~~

~~VFSLS (homogeneous)~~