Math 290 Monday, February 8 Section SS

RQ: #1 & #2

#3 LaTeX

$\{ \quad \}$ `\{` `\}`

$\langle \quad \rangle$ `\langle` `\rangle`

$\left\langle \vphantom{\frac{A}{B}} \right\rangle$ `\left\langle` `\right\rangle`

Tue - Problem Session

Thu - LI (concept!)

Fri - LDS

BYOB - Art

website: this semester's exam

Ex $S = \left\{ \begin{bmatrix}2\\3\\2\end{bmatrix}, \begin{bmatrix}-3\\0\\4\end{bmatrix}, \begin{bmatrix}-5\\6\\-8\end{bmatrix}, \begin{bmatrix}4\\-3\\6\end{bmatrix} \right\}$ $\quad \begin{bmatrix}0\\9\\-2\end{bmatrix} \in \langle S \rangle$?

shortcuts

$$\left(2\begin{bmatrix}2\\3\\2\end{bmatrix} + 6\begin{bmatrix}-3\\0\\4\end{bmatrix} + (-5)\begin{bmatrix}-5\\6\\8\end{bmatrix} + 7\begin{bmatrix}4\\-3\\6\end{bmatrix} \in \langle S \rangle \right)$$

Are there scalars a_1, a_2, a_3, a_4 so that

$$a_1\begin{bmatrix}2\\3\\2\end{bmatrix} + a_2\begin{bmatrix}-3\\0\\4\end{bmatrix} + a_3\begin{bmatrix}-5\\6\\-8\end{bmatrix} + a_4\begin{bmatrix}4\\-3\\6\end{bmatrix} = \begin{bmatrix}0\\9\\-2\end{bmatrix} ?$$

Definition of Span

Is there a solution to system w/ augmented matrix:

$$\left[\begin{array}{cccc|c} 2 & -3 & -5 & 4 & 0 \\ 3 & 0 & 6 & -3 & 9 \\ 2 & 4 & -8 & 6 & 2 \end{array}\right] ?$$

By Theorem SLSLC

RREF

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 3 \\ 0 & 1 & 3 & -2 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

not a pivot column

$\Rightarrow$ system consistent

RCLS

Answer: Yes

Ex Is $\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \in \langle S \rangle$?

① a_1, a_2, a_3, a_4 so that $a_1 \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix} + a_2 \begin{bmatrix} -3 \\ 0 \\ 4 \end{bmatrix} + a_3 \begin{bmatrix} -5 \\ 6 \\ -8 \end{bmatrix} + a_4 \begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$?

Defn of Span

② Solution to system $\left[\begin{array}{cccc|c} 2 & -3 & -5 & 4 & -1 \\ 3 & 0 & 6 & -3 & 2 \\ 2 & 4 & -8 & 6 & 4 \end{array}\right]$? Theorem SLSLC

③ $\xrightarrow{\text{RREF}}$ $\left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$

④ last column pivot $\xrightarrow{\text{RCLS}}$ inconsistent, no such scalars

$\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} \notin \langle S \rangle$

1 System w/ 5 equations, 7 variables, $\longrightarrow$ =-

$$\left[\begin{array}{ccccccc|c} 1 & 0 & 6 & 0 & -7 & 5 & 0 & -3 \\ 0 & 1 & 3 & 0 & 9 & -4 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 & 8 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

x_1, x_2, x_3, x_7 dependent

$D = \{1, 2, 4, 7\}$

$F = \{3, 5, 6, 8\}$ not a pivot $\Rightarrow$ consistent RCLS

(1) Null space of coefficient matrix (last column now zeros)

$x_3 \quad x_5 \quad x_6$ free

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} -3 - 6x_3 + 7x_5 - 5x_6 \\ 2 - 3x_3 - 9x_5 + 4x_6 \\ x_3 \\ 1 - 0x_3 - 2x_5 - 8x_6 \\ x_5 \\ x_6 \\ 3 - 0x_3 - 0x_5 - 0x_6 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -6 \\ -3 \\ \boxed{1} \\ -0 \\ \boxed{0} \\ \boxed{0} \\ -0 \end{bmatrix} + x_5 \begin{bmatrix} 7 \\ -9 \\ \boxed{0} \\ -2 \\ \boxed{1} \\ \boxed{0} \\ -0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 4 \\ \boxed{0} \\ -8 \\ \boxed{0} \\ \boxed{1} \\ -0 \end{bmatrix}$$

D = "nice pattern of zeros & ones"

VFSLS

$$N(A) = \left\langle \left\{ \begin{bmatrix} -6 \\ -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ -9 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 4 \\ 0 \\ -8 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\} \right\rangle$$

First, Second, Third, Fourth

(2) Original system (not null space)

Solution: $x_3 = x_5 = x_6 = 0$

Then $\underset{\sim}{x} = \begin{bmatrix} -3 \\ 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 3 \end{bmatrix}$

Theorem $\boxed{\text{PS}}\boxed{\text{PHS}}$