

Math 290

Friday, January 22

Section RREF

$$\begin{bmatrix} 2 & 3 & -6 \\ 4 & 9 & 3 \end{bmatrix} \begin{array}{l} 2 \text{ rows} \\ 3 \text{ columns} \end{array}$$

matrix

$$\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

vector, column vector

Ex Linear system

$$5x_1 + 6x_2 - 3x_3 = 8$$

$$9x_1 + x_2 + 4x_3 = 2$$

LS(A, b)

Augmented matrix of a system

$$\left[\begin{array}{ccc|c} 5 & 6 & -3 & 8 \\ 9 & 1 & 4 & 2 \end{array} \right]$$

representation of the system

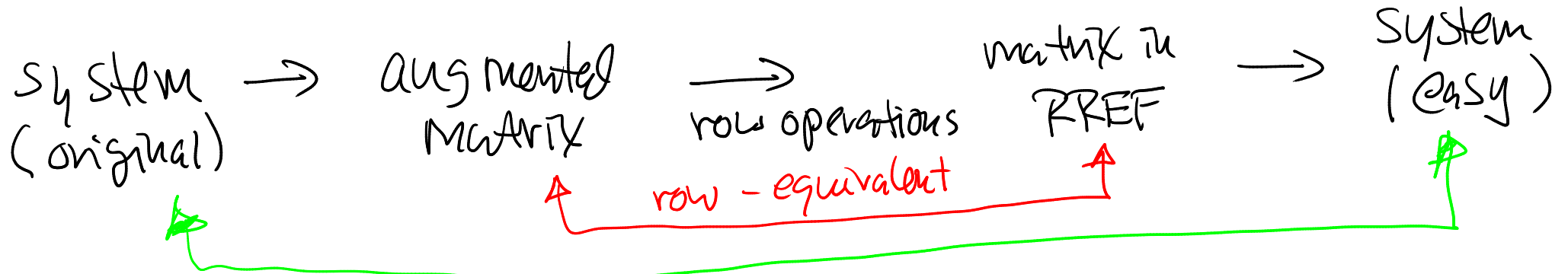
$$A = \begin{bmatrix} 5 & 6 & -3 \\ 9 & 1 & 4 \end{bmatrix} \quad \tilde{b} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

coefficient matrix vector of constants

Row Operations

- 1) Swap rows
- 2) Multiply row by non-zero scalar
- 3) Add a multiple of a row to another

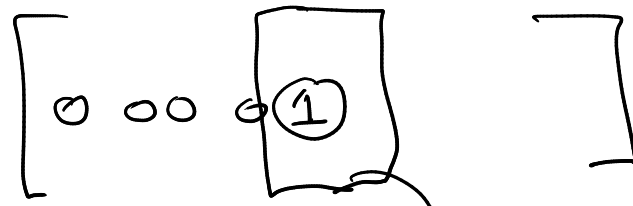
Defn Two matrices are row-equivalent if one can be obtained from another by row operations



RREF - always possible

UNIQUE
(theorem)

Pivot columns & leading ones



Pivot columns
 $\begin{matrix} \swarrow \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \\ \circ \end{matrix}$

equivalent systems

Solve $x_1 + 3x_2 + x_3 = 3$
 $x_1 + 2x_2 = 2$
 $2x_1 + 4x_2 + x_3 = 6$

REF (yesterday)
 ← pivot column

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 1 & 2 & 0 & 2 \\ 2 & 4 & 1 & 6 \end{array} \right] \xrightarrow{\substack{-R_1 + R_2 \\ -2R_1 + R_3}}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -2 & -1 & 0 \end{array} \right] \xrightarrow{-1R_2}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & 1 & -1 & 1 \\ 0 & -2 & -1 & 0 \end{array} \right]$$

$$\xrightarrow{\substack{-3R_2 + R_1 \\ 2R_2 + R_3}} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

↑ ↑
 pivot columns

$$\xrightarrow{\substack{-R_3 + R_2 \\ 2R_3 + R_1}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

↑ ↑ ↑
 pivot columns

system $\begin{cases} x_1 = 4 \\ x_2 = -1 \\ x_3 = 2 \end{cases}$

one solution only