Math 290 Friday, January 22 Section RREF

$$\begin{bmatrix} 2 & 3 & -6 \\ 4 & 9 & 3 \end{bmatrix}$$ 2 rows, 3 columns

matrix

$$\begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

vector, column vector

Ex Linear system

$LS(A, \underset{\sim}{b})$

$$5x_1 + 6x_2 - 3x_3 = 8$$
$$9x_1 + x_2 + 4x_3 = 2$$

same

$$A = \begin{bmatrix} 5 & 6 & -3 \\ 9 & 1 & 4 \end{bmatrix} \quad \underset{\sim}{b} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

coefficient matrix

vector of constants

Augmented matrix of a system

$$\left[\begin{array}{ccc|c} 5 & 6 & -3 & 8 \\ 9 & 1 & 4 & 2 \end{array}\right]$$

representation of the system

Row Operations

1) Swap rows
2) Multiply row by nonzero scalar
3) Add a multiple of a row to another

Defn Two matrices are row-equivalent if one can be obtained from another by row operations

system (original) $\rightarrow$ augmented matrix $\xrightarrow{\text{row operations}}$ matrix in RREF $\rightarrow$ system (easy)

row - equivalent

equivalent

RREF - always possible

UNIQUE (theorem)

Pivot columns & leading ones

$$\begin{bmatrix} 0 & 0 & 0 & 0 & \boxed{1} & & \end{bmatrix}$$

pivot columns

$$\begin{matrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{matrix}$$

RREF (yesterday)

Ex Solve

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 3 \\ x_1 + 2x_2 \quad\;\; &= 2 \\ 2x_1 + 4x_2 + x_3 &= 6 \end{aligned}$$

pivot column

$$\left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 1 & 2 & 0 & 2 \\ 2 & 4 & 1 & 6 \end{array}\right] \xrightarrow[-2R_1+R_3]{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & -1 & -1 & -1 \\ 0 & -2 & -1 & 0 \end{array}\right] \xrightarrow{-1R_2} \left[\begin{array}{ccc|c} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & -2 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[2R_2+R_3]{-3R_2+R_1} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow[2R_3+R_1]{-R_3+R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right] \xrightarrow{\text{system}} \begin{aligned} x_1 &= 4 \\ x_2 &= -1 \\ x_3 &= 2 \end{aligned}$$

pivot columns

pivot columns

equivalent systems

<u>one</u> solution <u>only</u>