Math 181 Thursday, April 29 Section 11.2

Fri – Problems
Housekeeping
BYOB – Summer
Mon – Snow Day
Tue – Exam 4
Chaps 8, 9, 11

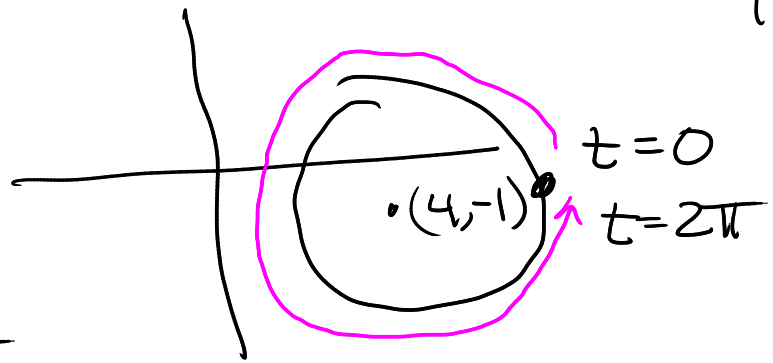
Arc Length

$(x(t), y(t))$ $\quad ds = \sqrt{x'(t)^2 + y'(t)^2}\, dt$

$$S = \int_{t=t_0}^{t=t_1} \sqrt{x'(t)^2 + y'(t)^2}\, dt$$

Ex $\quad x(t) = 4 + 2\cos(t)$

$y(t) = -1 + 2\sin(t)$



Circumference : $t = 0, t = 2\pi$

$x'(t) = -2\sin(t)$

$y'(t) = 2\cos(t)$

$$ds = \sqrt{(-2\sin(t))^2 + (2\cos(t))^2}\, dt$$

$$= \sqrt{4\sin^2(t) + 4\cos^2(t)}\, dt$$

$$= 2\sqrt{\sin^2(t) + \cos^2(t)}\, dt$$

$$= 2\, dt$$

$$S = \int_0^{2\pi} 2\, dt$$

$$= 2t\Big|_0^{2\pi}$$

$$= 2(2\pi) - 2\cdot 0$$

$$= 2\pi(2)$$

Ex $y^2 = x^3$, length from $x=1$ to $x=2$?

$y = \pm\sqrt{x^3} = \pm x^{3/2}$

$ds = \sqrt{1+f'(x)^2}\,dx = \sqrt{1+(\pm 3/2\, x^{1/2})^2}\,dx$

$= \sqrt{1+\tfrac{9}{4}x}\,dx$ "easy" to integrate

OR parametrize the curve.

A parameterization is $x(t)=t^2$ & $y(t)=t^3$.

$x=1$, then $t=1$; $x=2$ then $t=\sqrt{2}$ (so $y>0$)

$x'(t) = 2t$

$y'(t) = 3t^2$

$ds = \sqrt{(2t)^2+(3t^2)^2}\,dt$

$= \sqrt{4t^2+9t^4}\,dt$

$= \sqrt{t^2(4+9t^2)}\,dt$

$= t\sqrt{4+9t^2}\,dt$

$s = \int_{t=1}^{t=\sqrt{2}} t\sqrt{4+9t^2}\,dt$

$u = 4+9t^2$

$du = 18t\,dt$

$\frac{1}{18}du = t\,dt$

$= \int_{t=1}^{t=\sqrt{2}} \frac{1}{18}u^{1/2}\,du$

Erdös

Length function

$$s(t) = \int_{t_0}^{t} ds = \int_{u=t_0}^{u=t} \sqrt{x'(u)^2 + y'(u)^2}\, du$$

$s(t)$: length, time

$$\text{speed} = \frac{ds}{dt} = s'(t) = \frac{d}{dt} \int_{u=t_0}^{u=t} \sqrt{x'(t)^2 + y'(u)^2}\, du$$

FTC $\Rightarrow$ $= \sqrt{x'(t)^2 + y'(t)^2}$

(OR, $ds = \sqrt{x'(t)^2 + y'(t)^2}\, dt$

$\Rightarrow \frac{ds}{dt} = \sqrt{x'(t)^2 + y'(t)^2}$)

Ex Line: $x(t) = 2t + 3$, $y(t) = 5t + 3$ speed at $t = 1$ $\quad t=1 \Rightarrow (5,8)$

$$\frac{ds}{dt} = \sqrt{2^2 + 5^2} = \sqrt{29}$$ ← constant speed, so $\left.\frac{ds}{dt}\right|_{t=1} = \sqrt{29}$

Same line, new parametrization

$x(t) = 2t^2 + 3$
$y(t) = 5t^2 + 3$

(eliminate parameter, get the same line as ★)

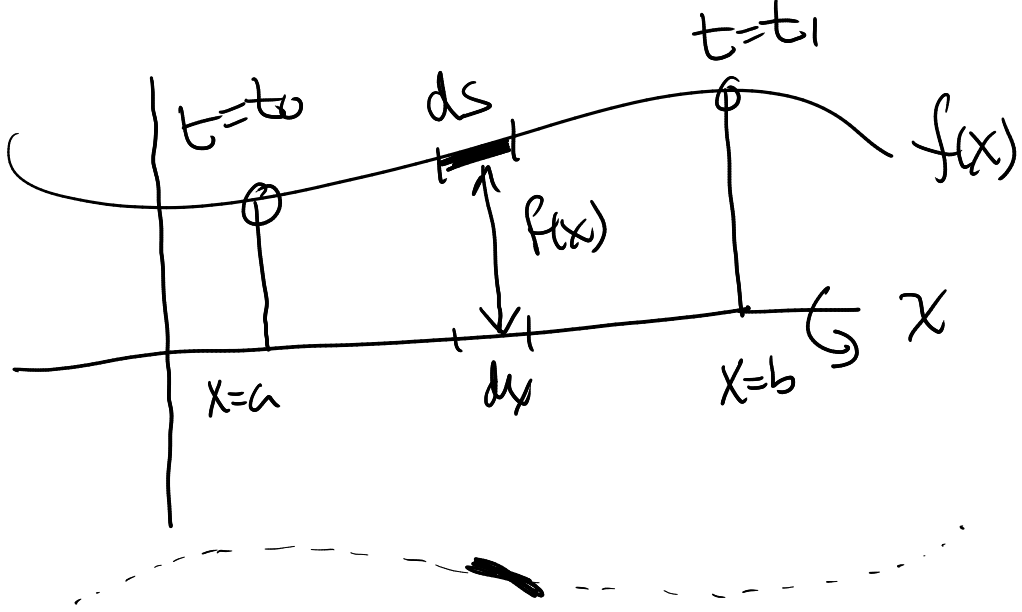
$$\frac{ds}{dt} = \sqrt{(4t)^2 + (10t)^2}$$
$$= \sqrt{16t^2 + 100t^2}$$
$$= \sqrt{116t^2}$$
$$= \sqrt{116}\,t$$ ← not constant

$$\left.\frac{ds}{dt}\right|_{t=1} = \sqrt{116} \cdot 1 = \sqrt{116}$$

$t=1 \Rightarrow (5,8)$

different

Surface area (revolution)



$$dA = \underbrace{2\pi f(x)}_{\text{circumference}} \underbrace{ds}_{\text{width}}$$

Parametric Curve

radius y

$$dA = 2\pi y \, ds$$

$$= 2\pi y(t) \sqrt{x'(t)^2 + y'(t)^2} \, dt$$