Math 181 Monday, April 19 Section 9.2

Tue - 9.3

Thu - 9.3/9.4

Fri - 9.4

BYOB - Free Play

WW 9.1.2

$8 \ln y \, y' - ty = 0$

$8 \ln y \frac{dy}{dt} - ty = 0$

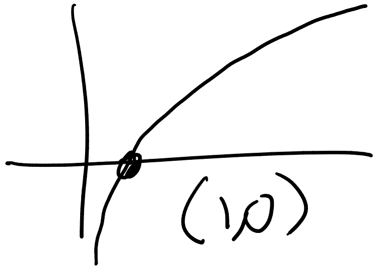
$8 \ln y \frac{dy}{dt} = ty$

$8 \ln y \frac{1}{y} dy = t\,dt$

$\int 8 \ln y \frac{1}{y} dy = \int t\,dt$

$u = \ln y$

$du = \frac{1}{y} dy$



$\int 8u\,du = \int t\,dt$

$4u^2 = t^2/2 + C$

$(\ln y)^2 = \frac{1}{8}t^2 + C$

$\ln y = \pm\sqrt{\frac{1}{8}t^2 + C}$

$y>1 \quad \ln y > 0 \quad \ln y = +\sqrt{\frac{1}{8}t^2+C} \quad y = e^{\sqrt{\frac{1}{8}t^2+C}}$

$y<1 \quad \ln y < 0 \quad \ln y = -\sqrt{\frac{1}{8}t^2+C} \quad y = e^{-\sqrt{\frac{1}{8}t^2+C}}$

WW 9.1.3

$$\frac{dx}{dt} \Rightarrow x(t) \qquad x(0)=4$$

$$\int \frac{1}{x^2+16}\,dx = \int \frac{1}{t^2+16}\,dt$$

$$\frac{1}{4}\arctan\left(\frac{x}{4}\right) = \frac{1}{4}\arctan\left(\frac{t}{4}\right) + C$$

$$\frac{1}{4}\arctan\left(\frac{4}{4}\right) = \frac{1}{4}\arctan\left(\frac{0}{4}\right) + C$$

$$\frac{1}{4}\left(\frac{\pi}{4}\right) = \frac{1}{4}0 + C$$

$$\pi/16 = C$$

$$\longrightarrow \frac{1}{4}\arctan\left(\frac{x}{4}\right) = \frac{1}{4}\arctan\left(\frac{t}{4}\right) + \frac{\pi}{16}$$

$$\arctan\left(\frac{x}{4}\right) = \arctan\left(\frac{t}{4}\right) + \pi/4$$

$$\frac{x}{4} = \tan\left(\arctan\left(\frac{t}{4}\right) + \frac{\pi}{4}\right)$$

$$x = 4\tan\left(\arctan\left(\frac{t}{4}\right) + \pi/4\right)$$

Seperable Differential Equation (Exponential : $\frac{dy}{dt} = ky$)

$$\frac{dy}{dt} = k(y-b)$$

growth rate is proportional to amount different than b

$$\frac{1}{y-b}\,dy = k\,dt$$

$$\int \frac{1}{y-b}\,dy = \int k\,dt$$

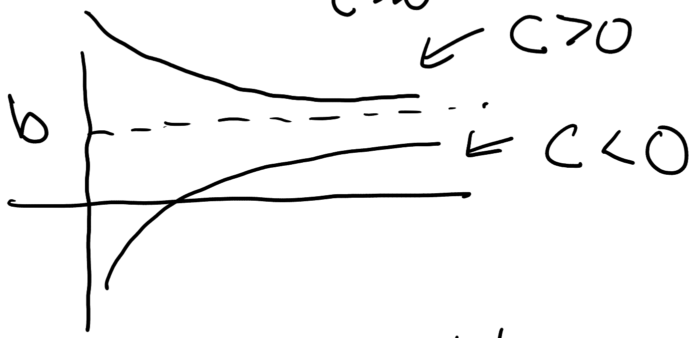
$$\ln(y-b) = kt + C$$

$$y-b = e^{kt+C}$$

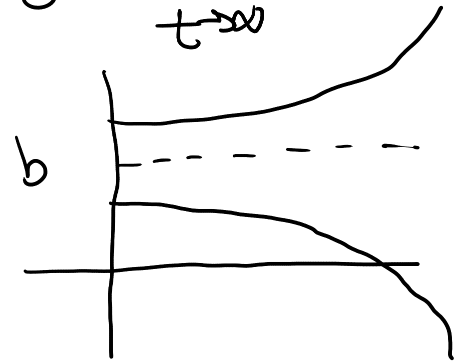
$$y = b + e^{kt}e^{C}$$

$$= b + Ce^{kt}$$

$k<0$, $\lim_{t\to\infty} e^{kt} = 0$



$k>0$ $\lim_{t\to\infty} e^{kt} = \infty$



Ex <u>Cooling</u> Temperature change of an object is proportional to difference in temperature of its surroundings.

95°C cup of coffee, surroundings at 20°

After 5 minutes, temperature is 87°.

When is your coffee at 21°? (Cold!)

$$\frac{dT}{dt} = k(T-20) \rightarrow T(t) = 20 + Ce^{kt}$$

$$95 = T(0) = 20 + Ce^{k(0)} = 20 + C \cdot 1 \Rightarrow C = 75$$

$$T(t) = 20 + 75e^{kt}$$

$$87 = T(5) = 20 + 75e^{k5}$$

$$67 = 75e^{5k}$$

$$67/75 = e^{5k}$$

$$\ln(67/75) = 5k$$

$$\frac{1}{5}\ln(67/75) = k$$

$$-0.0225 = k$$

Time when $T(t) = 21$?

$$21 = T(t) = 20 + 75 e^{-0.0225 t}$$

$$1 = 75 e^{-0.0225 t}$$

$$1/75 = e^{-0.0225 t}$$

$$\ln(1/75) = -0.0225 t$$

$$t = \frac{-1}{0.0225} \ln(1/75) = 191.85 \text{ minutes}$$

<u>Annuity</u> \$1,000,000 in savings, earns 7% interest,
Draw \$100,000 year. How long will this last?

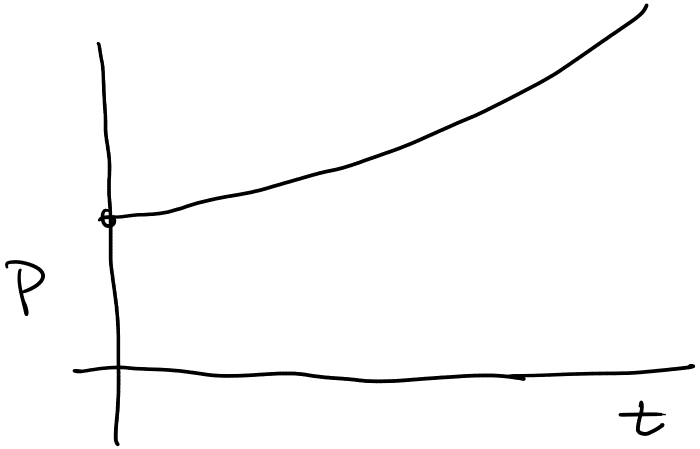
General, P = principal, r = interest rate (growth), N = annual payment

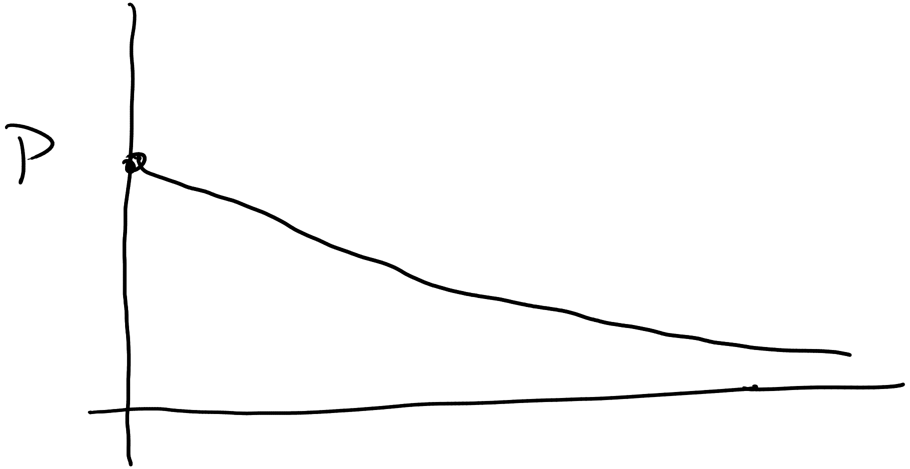
Behavior depends on $P - N/r$: $P - N/r > 0 \Rightarrow P > \frac{N}{r} \Rightarrow \underline{\underline{rP}} > \underline{\underline{N}}$

amount of interest   payment

$P - N/r < 0$, deplete savings

$P - N/r > 0$





$$P(t) = \frac{N}{r} + \left(P - \frac{N}{r}\right)e^{rt}$$

How long will \$100,000 withdrawals last?

$$0 = \frac{100{,}000}{.07} + \left(1{,}000{,}000 - \frac{100{,}000}{.07}\right)e^{.07t}$$

$t = 17$