Math 181 Friday, April 16 Section 9.1

Exponential Growth & Decay

Mon - 9.2
Tue - 9.3
Thu - 9.3/9.4
Fri - 9.4

Differential Equation

$$\frac{dy}{dx} = ky$$

constant: positive ⇒ growth
negative ⇒ decay

"rate of change is proportional the amount present"

Growth: money, people infected, population

Decay: radioactive, drugs in your body

Solve this:

(Separable)

$$\frac{1}{y}\,dy = k\,dx$$

$$\int \frac{1}{y}\,dy = \int k\,dx$$

$$\ln y = kx + C$$

$$e^{\ln y} = e^{kx+C}$$

$$y = e^{kx}e^{C} = Ce^{kx}$$

Ex Doubling time for money at interest rate r

$M(t) = P e^{rt}$ $\qquad \frac{dM}{dt} = rM$

$M(0) = Pe^{r(0)} = Pe^0 = P$

$M(t) = 2P$??? $\quad Pe^{rt} = 2P \Rightarrow e^{rt} = 2$

$\Rightarrow rt = \ln 2 \Rightarrow t = \frac{1}{r}\ln 2 = \frac{\ln 2}{r}$

Interest rate 7%, $r = 0.07$

Doubling time: $\frac{\ln 2}{.07} = \frac{100 \ln 2}{7} \cong \frac{69}{7} \approx \frac{70}{7} \approx 10$ years

Ex June 1998 world population 5,926,466,814

Sept 2006 world population 6,543,462,601

① Prediction for June 2020

② When hit 50 billion? (carrying capacity)

$P(t) = P_0 e^{kt}$ $\quad t=0 \iff$ June 1998

$5.926 = P(0) = P_0 e^{k\cdot 0} = P_0 e^0 = P_0 \cdot 1 = P_0$

$6.543 = P(98) = 5.926\, e^{k\frac{(98)}{12}} \Rightarrow \frac{6.543}{5.926} = e^{98k}$ based on years

(98 months)

$\Rightarrow 98k = \ln\left(\frac{6.543}{5.926}\right) \Rightarrow k = \frac{1}{98}\ln\left(\frac{6.543}{5.926}\right) = 0.0121271 \ (\gtrsim 1\%)$

$P(166) = 5.926\, e^{\frac{1}{98}\ln\left(\frac{6.543}{5.926}\right)\cdot\frac{166}{12}} = 5.926\, e^{\frac{166}{98}\ln\left(\frac{6.543}{5.926}\right)} = 7{,}730{,}630{,}173$

(166 = 2020 in months)

t for $P(t) = 50$

Solve $50 = 5.926\, e^{.0121t}$

$\frac{50}{5.926} = e^{.0121t} \Rightarrow .0121t = \ln\left(\frac{50}{5.926}\right)$

$t = \frac{1}{.0121}\ln\left(\frac{50}{5.926}\right) = 167.6$ years

Ex Stock market returns 7% per year over inflation, last 60 years.
Pay for financial management. ~ 1% years
You might do a lot of research, & do well w/ investments.
Compare P dollars at 6% vs P dollars at 8%
for 40 years.

1st case end up with $Pe^{.06(40)}$

2nd case end up with $Pe^{.08(40)}$

Ratio $$\frac{Pe^{.08(40)}}{Pe^{.06(40)}} = \frac{e^{.08(40)}}{e^{.06(40)}} = e^{.08(40) - .06(40)}$$

$$= e^{(.08-.06)40} = e^{(.02)(40)} = e^{.8} \doteq 2.225$$