Math 181 Thursday, March 25 Section 10.6

Fri – 10.6

Thu – 10.7

10.6 – preview

10.6 not Differential Equations

Two concepts:

1) Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

2) Approximate a function by a infinite polynomial

Ex Define a new function of x by

$$g(x) = \sum_{n=0}^{\infty} 3^n x^n$$

$$g(1/4) = \sum_{n=0}^{\infty} 3^n (1/4)^n = \sum_{n=0}^{\infty} (3/4)^n = \frac{1}{1-3/4} = 4$$

↑ geometric, $r = 3/4 < 1$

$$g(-2/7) = \sum_{n=0}^{\infty} 3^n (-2/7)^n = \sum_{n=0}^{\infty} (-1)^n (6/7)^n = \sum_{n=0}^{\infty} (-6/7)^n = \frac{1}{1-(-6/7)} = 7/13$$

alternating series

geometric, $|r| = |-6/7| < 1$

$$g(2) = \sum_{n=0}^{\infty} 3^n 2^n = \sum_{n=0}^{\infty} 6^n \quad \text{not defined; } g(2) \text{ is not defined}$$

geometric series, $|r| = |6| > 1$ the series diverges

$$g(0) = \sum_{n=0}^{\infty} 3^n \cdot 0^n = \sum_{n=0}^{\infty} 0 = 0$$

for what x

- when is $g(x)$ defined?
- For what x does the infinite series converge?

Ratio Test

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{3^{n+1} x^{n+1}}{3^n x^n} \right| = \lim_{n\to\infty} 3|x| = 3 \lim_{n\to\infty} |x| = 3|x|$$

Function defined $\iff$ series converges $\iff$ $L < 1 \iff 3|x| < 1 \iff |x| < 1/3 \iff$

$$-\frac{1}{3} < x < 1/3$$

$x = \frac{1}{3}$ $\quad g(1/3) = \sum_{n=0}^{\infty} 3^n (1/3)^n = \sum_{n=0}^{\infty} 1$ diverges

$x = -\frac{1}{3}$ $\quad g(-1/3) = \sum_{n=0}^{\infty} 3^n (-1/3)^n = \sum_{n=0}^{\infty} (-1)^n$ $\quad \lim_{n\to\infty} (-1)^n = \text{DNE} \neq 0$ oscillates

diverges by nth term test for divergence

So $g(x)$ defined only for $\underbrace{-1/3 < x < 1/3}_{\text{interval of convergence}}$ absolute convergence.

radius of convergence is $1/3$

$\boxed{\text{what does } g(x) \text{ converge to?}}$

$$g(x) = \sum_{n=0}^{\infty} 3^n x^n = \sum_{n=0}^{\infty} (3x)^n = \frac{1}{1-3x}$$

geometric w/ $r = 3x$