Math 181 Friday, March 19 Section 10.3

Mon 10.4

Tue, Thu, Fri 10.5/10.6

Integral Test

$$\sum_{n=1}^{\infty} a_n \text{ converges} \iff \int_1^{\infty} a(x)\,dx \text{ converges}$$

<u>Fact</u> (p-series) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p>1$

diverges if $p\le 1$

<u>Proof</u> by Integral Test

<u>Ex</u> $\sum_{n=1}^{\infty} \frac{1}{n^3}$ converges

<u>Ex</u> $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges

<u>Ex</u> $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, $p=1$ Harmonic series

Comparison Test $\sum_{n=1}^{\infty} a_n$ with $a_n \geq 0$

1) If $b_n \geq a_n$ & $\sum_{n=1}^{\infty} b_n$ converges $\Rightarrow$ $\sum_{n=1}^{\infty} a_n$ converges

2) If $c_n \leq a_n$ & $\sum_{n=1}^{\infty} c_n$ diverges $\Rightarrow$ $\sum_{n=1}^{\infty} a_n$ diverges

Ex $\sum_{n=1}^{\infty} \frac{2n-3}{4n^3+6n+2}$ looks like this converges, kinda like $\sum \frac{1}{n^2}$

Comparison $\frac{2n-3}{4n^3+6n+2} < \frac{2n}{4n^3} = \frac{1}{2n^2} < \frac{1}{n^2}$ (base sequence)

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges ($p=2>1$) so by the comparison test $\sum_{n=1}^{\infty} \frac{2n-3}{4n^3+6n+2}$ converges (also).

Ex $\sum_{n=1}^{\infty} \frac{3^n+n}{2^n-n^2}$

$\frac{3^n+\cancel{n}}{2^n-\cancel{n^2}} \approx \frac{3^n}{2^n} = \left(\frac{3}{2}\right)^n$ diverges?

Comparison: $\frac{3^n+n}{2^n-n^2} > \frac{3^n}{2^n} = \left(3/2\right)^n$

Know $\sum_{n=1}^{\infty} (3/2)^n$ diverges because geometric, $r = 3/2 > 1$, so by comparison $\sum_{n=1}^{\infty} \frac{3^n+n}{2^n-n^2}$ diverges.

n-th term test

Theorem $\sum_{n=1}^{\infty} a_n$? If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges.

DONUTS

Ex $\sum_{n=1}^{\infty} \frac{n^2+n}{3n^2+6n+2}$

$\frac{n^2+\cancel{n}}{3n^2+\cancel{6n}+\cancel{2}} = \frac{n^2}{3n^2} = \frac{1}{3}$

$\lim_{n\to\infty} \frac{n^2+n}{3n^2+6n+2} = \frac{1}{3} \neq 0$

So by nth-term test for divergence, we see $\sum_{n=1}^{\infty} \frac{n^2+n}{3n^2+6n+2}$ diverges

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ $\qquad \displaystyle\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0 \leftarrow$ Hopeful!, but this series does converge.

Limit Form of Comparison Test

Ex $\displaystyle\sum_{n=1}^{\infty} \frac{8n^3+6n^2+n}{4n^6-8n^5+12}$

$$\frac{8n^3+\cancel{6n^2}+\cancel{n}}{4n^6-\cancel{8n^5}+\cancel{12}} = \frac{8n^3}{4n^6} = \frac{2}{n^3}$$ converges

$$\lim_{n\to\infty} \frac{\left(\frac{8n^3+6n^2+n}{4n^6-8n^5+12}\right)}{1/n^2} = \lim_{n\to\infty} \frac{8n^5+6n^4+n^3}{4n^6-8n^5+12} = 0$$

forms a convergent series $\rightarrow$ $1/n^2$

$\Rightarrow$ original series converges also