Math 181 Thursday, March 18 Section 10.3

Fri 10.3/10.4
BYOB - Pets
Mon - 10.4
Tue-Fri 10.5/10.6

Infinite series

$$\sum_{n=0}^{\infty} a_n = ?$$

"base" sequence

Partial sums

$S_0 = a_0$

$S_1 = a_0 + a_1$

$S_2 = a_0 + a_1 + a_2$

$S_3 = a_0 + a_1 + a_2 + a_3$

$S_4 \quad a_0 + a_1 + a_2 + a_3 + a_4$

the "sequence of partial sums"

sums of terms of the base sequence

finite sums

Defn

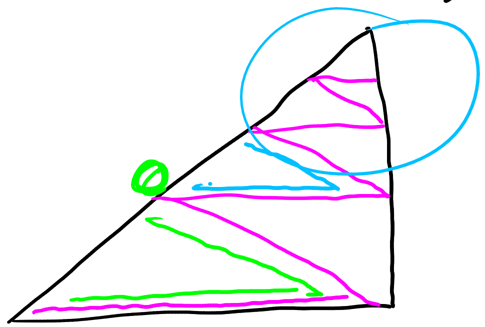
$$\sum_{n=0}^{\infty} a_n = \lim_{n\to\infty} S_n$$

Question: Does this limit even exist?

Yes - series converges.

No - series diverges.

WW 10.2.6 Length of zig-zag path?



$$\sum = 12 + \frac{12}{\sqrt{2}}$$

12

$$\text{Length} = \left(12 + \frac{12}{\sqrt{2}}\right) + \left(12 + \frac{12}{\sqrt{2}}\right)\left(\frac{1}{2}\right) + \left(12 + \frac{12}{\sqrt{2}}\right)\left(\frac{1}{2}\right)^2 + \left(12 + \frac{12}{\sqrt{2}}\right)\left(\frac{1}{2}\right)^3 + \cdots\cdots$$

$$= \left(12 + \frac{12}{\sqrt{2}}\right)\left(1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \cdots\right)$$

geometric series

$$= \left(12 + \frac{12}{\sqrt{2}}\right) \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$$

converges to $\frac{1}{1-1/2}$

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r} \quad \text{if } |r| < 1$$

$$= \left(12 + \frac{12}{\sqrt{2}}\right)\left(\frac{1}{1-1/2}\right) = 2\left(12 + \frac{12}{\sqrt{2}}\right)$$

Geometric Series, Gauss (~1800's)

$1+2+4+8+16+32+\cdots = \sum_{n=0}^{\infty} 2^n = \frac{1}{1-2} = -1 \qquad |2| \not< 1$

When a geometric series converges, we can determine what it converges to. (That is the exception.)

Theorem $a_n \geq 0$, $\sum_{n=1}^{\infty} a_n$ converges if and only if $\int_1^{\infty} a(x)\,dx$ converges

"Integral Test"

Ex $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges "Harmonic Series"

$a(x) = \frac{1}{x}$

$\int_1^{\infty} \frac{1}{x}\,dx = \lim_{b\to\infty} \ln(b) - \ln(1)$

$=$ does not exist, unbounded, diverges

Ex $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges

$a(x) = \frac{1}{x^2} \qquad \int_1^{\infty} \frac{1}{x^2}\,dx$

$= \lim_{b\to\infty} -\frac{1}{b} - \left(-\frac{1}{1}\right) = 0 + 1 = 1$

Integral converges $\Rightarrow$ series converges (to $\frac{\pi^2}{6}$)