Math 181 Monday, March 8

Section 7.8 (part 2)

Tue 10.1

Thu 10.1 / 10.2

Fri 10.2

BYOB TV shows

$\int e^{-x^2} dx$ nobody knows

Can't use FTC on $\int_1^5 e^{-x^2} dx$; has no exact answer.

Next best thing: <u>approximate</u>.

Simpson's Rule: "quadratic tops"

$$S_n = \frac{1}{3}\left(f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right)\Delta x$$ <u>n even</u>

<u>Ex</u> $\int_1^5 e^{-x^2} dx$ $S_8 = 0.139015$

$\Delta x = \frac{5-1}{8} = \frac{1}{2}$

$S_{100} = 0.139402$

Error Bounds

Trapezoidal Rule: Estimate $\int_a^b f(x)\,dx$, w n points. Let $M = \max_{[a,b]} |f''(x)|$

Then
$$\underbrace{\left| \underset{\uparrow\,\text{approximation}}{T_n} - \underset{\uparrow\,\text{exact}}{\int_a^b f(x)\,dx} \right|}_{\text{error}} \underset{\uparrow\,\underline{\underline{\text{inequality}}}}{\le} \underbrace{\frac{(b-a)^3}{12n^2} M}_{\uparrow\,\text{"upper bound"}}$$

Simpson's Rule: Estimate $\int_a^b f(x)\,dx$ w/ n points. Let $M = \max_{[a,b]} |f^{(4)}(x)|$

Then
$$\left| S_n - \int_a^b f(x)\,dx \right| \le \boxed{\frac{(b-a)^5}{180n^4} M}$$

Ex $\int_1^5 e^{-x^2}\,dx$ Plot in Sage suggests $|f^{(4)}(x)| \le \overset{\text{could be 8, maybe 7.5?}}{10}$ on $[1,5]$

$$\text{error} \le \frac{(5-1)^5\ 10}{180\ \underset{\uparrow\, n=8}{8}^4} = 1.3 \times 10^{-2} \quad \Bigg\| \quad \text{error} \le \frac{(5-1)^5 \cdot 10}{180\,(100)^4} = 5.6 \times 10^{-7}$$

Ex Compute $\int_1^5 e^{-x^2}\,dx$ to an accuracy of 10^{-5} using Trapezoidal Rule.

What n? Plot of $|f^{(2)}(x)|$ on $[1,5]$ suggests $M=1$.

Want
$$\frac{(5-1)^3\,1}{12\,n^2} \le 10^{-5}$$

$$\frac{4^3}{12} \le 10^{-5}\,n^2$$

$$10^5\,\frac{4^3}{12} \le n^2$$

$$\sqrt{10^5\,\frac{4^3}{12}} \le n \qquad (\sqrt{\ } \text{ is increasing function})$$

$$\underline{730.26} \le n$$