Math 181 Thursday, March 4 Section 7.7 (part 2)

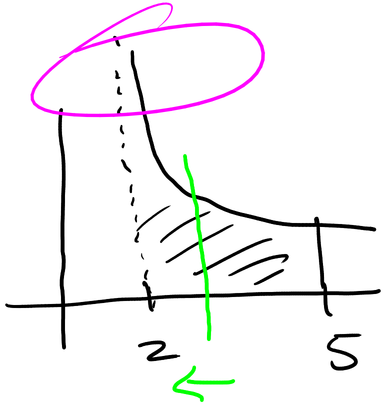
Fri 7.8
BYOB - solo outdoor sport

Mon 7.8

Tue 10.1

Vertically Improper

Ex $\displaystyle\int_{x=2}^{x=5} \frac{x\,dx}{\sqrt{x^2-4}}$



$\displaystyle\int_{2.1}^{5} \frac{x\,dx}{\sqrt{x^2-4}} = 3.942$

$\displaystyle\int_{2.01}^{5} \frac{x\,dx}{\sqrt{x^2-4}} = 4.382$

$\displaystyle\int_{2.0001}^{5} \frac{x\,dx}{\sqrt{x^2-4}} = 4.562575$

one-sided limit "approach from the right"

$$\int_2^5 \frac{x\,dx}{\sqrt{x^2-4}} = \lim_{a\to 2^+} \int_a^5 \frac{x\,dx}{\sqrt{x^2-4}}$$

$u = x^2-4$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$= \lim_{a\to 2^+} \int_a^5 \frac{\frac{1}{2}du}{u^{1/2}}$$

$$= \frac{1}{2}\lim_{a\to 2^+} \left.\frac{u^{1/2}}{1/2}\right|_a^5 = \lim_{a\to 2^+} \left.(x^2-4)^{1/2}\right|_a^5$$

$$= \lim_{a\to 2^+} (5^2-4)^{1/2} - (a^2-4)^{1/2} = 21^{1/2} - 0^{1/2}$$

$$= \sqrt{21} = 4.582575695$$

Ex $\int_{-2}^{2} \frac{1}{x^2}\,dx$ (not -1, like on Tuesday)

$$= \int_{-2}^{0} \frac{1}{x^2}\,dx + \int_{0}^{2} \frac{1}{x^2}\,dx = \lim_{b \to 0^-} \int_{-2}^{b} \frac{1}{x^2}\,dx + \lim_{a \to 0^+} \int_{a}^{2} \frac{1}{x^2}\,dx$$

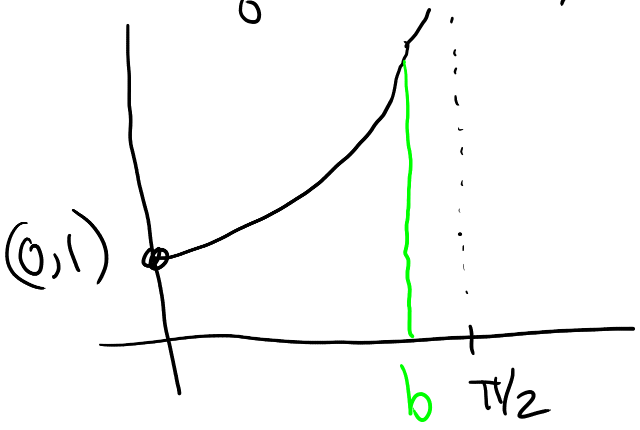
$$= \lim_{b \to 0^-} \left. \frac{x^{-1}}{-1} \right|_{-2}^{b} + \lim_{a \to 0^+} \left. \frac{x^{-1}}{-1} \right|_{a}^{2}$$

$$= \lim_{b \to 0^-} \boxed{-\frac{1}{b}} - \left(-\frac{1}{-2}\right) + \lim_{a \to 0^+} -\frac{1}{2} \boxed{- \left(-\frac{1}{a}\right)} \qquad (= -\infty + \infty)$$

= does not exist (unbounded) + does not exist (unbounded)

= does not exist

Ex $\int_0^{\pi/2} \sec(x)\,dx$



$$= \lim_{b \to \pi/2^-} \int_0^b \sec(x)\,dx$$

$$= \lim_{b \to \pi/2^-} \ln(\sec(x) + \tan(x)) \Big|_0^b$$

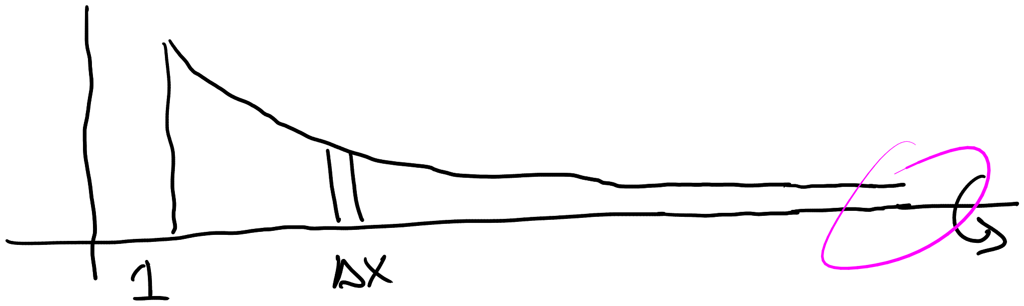
$$= \lim_{b \to \pi/2^-} \ln(\underbrace{\overset{\uparrow}{\sec(b)}}_{\text{unbounded positive}} + \underbrace{\overset{\uparrow}{\tan(b)}}_{\text{unbounded positive}}) - \ln(\sec(0) + \tan(0))$$

$\ln(\text{huge}) = \text{huge}$

Does not exist

Ex Gabriel's Horn

Rotate $y = 1/x$, $1 \le x < \infty$ around x-axis.



Volume: $\displaystyle\int_{x=1}^{x=\infty} \pi\left(\frac{1}{x}\right)^2 dx$

$= \lim_{b\to\infty} \pi \int_1^b \frac{1}{x^2}\,dx$

$= \pi \lim_{b\to\infty} \left.\frac{x^{-1}}{-1}\right|_1^b = \pi \lim_{b\to\infty} \frac{-1}{b} - \left(-\frac{1}{1}\right)$

$= \pi(-0+1) = \pi$

Surface Area

$\displaystyle\int 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$

$f'(x) = \frac{d}{dx}\frac{1}{x} = -1x^{-2}$

$\displaystyle= \int_{x=1}^{x=\infty} 2\pi \frac{1}{x}\sqrt{1+(-1x^{-2})^2}\,dx = \int_{x=1}^{x=\infty} 2\pi \frac{1}{x}\sqrt{1+\frac{1}{x^4}}\,dx = \int_{x=1}^{x=\infty} 2\pi\frac{1}{x}\sqrt{\frac{x^4+1}{x^4}}\,dx$

$\displaystyle= \int_{x=1}^{\infty} 2\pi \frac{1}{x}\frac{1}{x^2}\sqrt{x^4+1}\,dx = \int_{x=1}^{x=\infty} 2\pi\frac{x}{x^4}\sqrt{x^4+1}\,dx$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$= \int_{x=1}^{x=\infty} 2\pi \frac{\frac{1}{2}}{u^2}\sqrt{u^2+1}\,du > \int_{x=1}^{\infty} \pi \frac{1}{u^2}\sqrt{u^2}\,du = \int_{x=1}^{x=\infty} \pi \frac{1}{u}\,du$$

$$= \lim_{b\to\infty} \int_1^b \pi \frac{1}{u}\,du = \lim_{b\to\infty} \pi \ln u \Big|_1^b = \lim_{b\to\infty} \pi \ln(x^2)\Big|_1^b$$

$$= \lim_{b\to\infty} \pi\left(\ln b^2 - \ln 1^2\right)$$ does not exist, unbounded

So.... unbounded surface area, finite volume.

Put π gallons of paint into the horn, but it wouldn't be enough paint to paint the outside of the horn. A paradox.