Math 181 Friday, February 26 Section 7.4

Mon - 7.5

Tue - 7.5/7.7

Thu - 7.7

Fri - 7.8

Mon - 7.8

<u>WW</u> Preview ✓

#5, #6 complete the square

#5 $x^2+4x+13 = (x^2+4x+4)+9$

$= (x+2)^2+3^2$

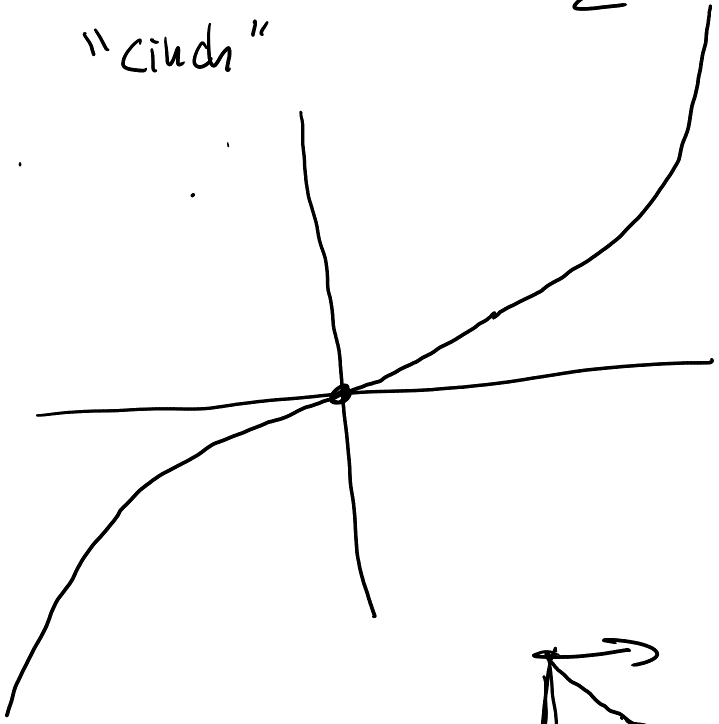
$x+2 = 3\tan\theta$

#7 Solid of Revolution

Hyperbolic Functions

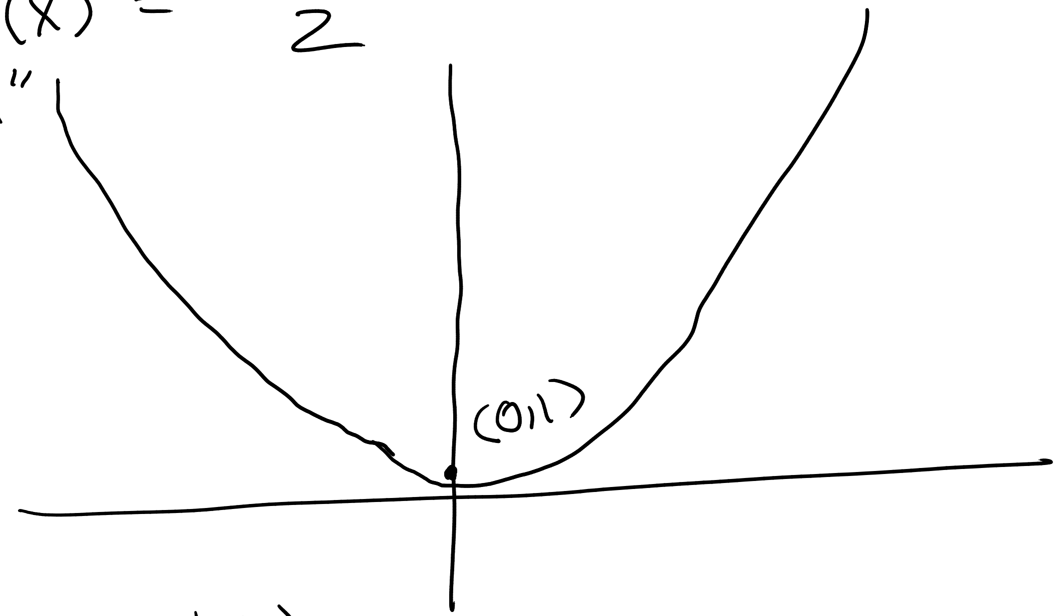
$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

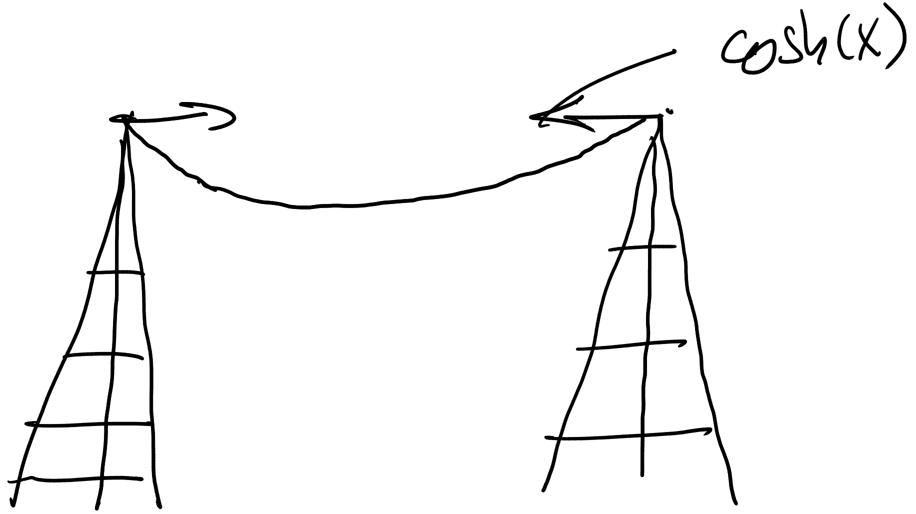
"cinch"



$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

"ko-sh"





Fundamental Identity

$$\cosh^2(t) - \sinh^2(t) = \left(\frac{e^t + e^{-t}}{2}\right)^2 - \left(\frac{e^t - e^{-t}}{2}\right)^2$$

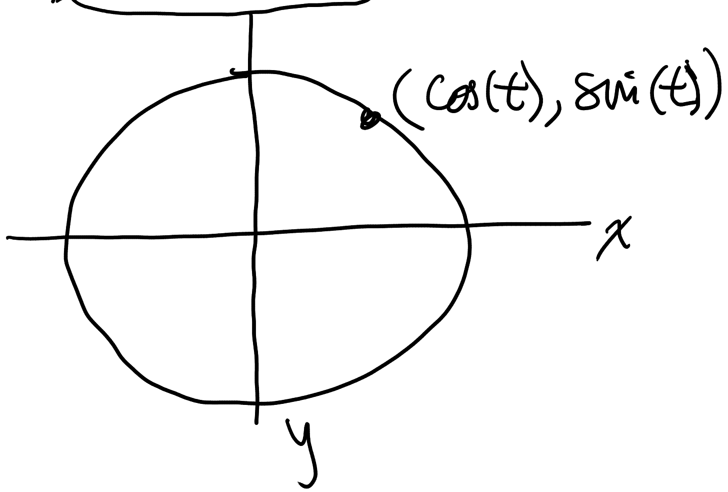
$$= \frac{1}{4}\left[\cancel{(e^t)^2} + 2e^t e^{-t} + \cancel{(e^{-t})^2} - \left(\cancel{(e^t)^2} - 2e^t e^{-t} + \cancel{(e^{-t})^2}\right)\right]$$

$$= \frac{1}{4}\left[2e^0 - (-2e^0)\right] = \frac{1}{4}\left[2+2\right] = 1$$

$x = \sin(t)$ $\quad y = \cos(t) \Rightarrow x^2 + y^2 = 1$

$x^2 + y^2 = 1$

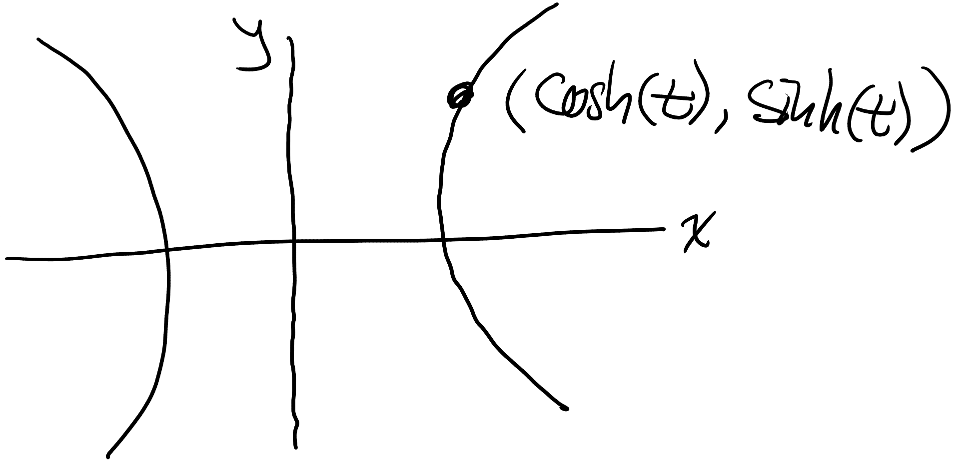
Circle



$x = \cosh(t)$, $y = \sinh(t) \Rightarrow x^2 - y^2 = 1$

$x^2 - y^2 = 1$

hyperbola



Other Facts: $\sinh(2x) = 2\sinh(x)\cosh(x)$ & MORE

$\frac{d}{dx}\sinh(x) = \cosh(x)$ $\frac{d}{dx}\cosh(x) = \sinh(x)$

$\frac{d}{dx}\tanh(x) = \operatorname{sech}^2(x)$ $\frac{d}{dx}\coth(x) = -\operatorname{csch}^2(x)$

$\frac{d}{dx}\operatorname{sech}(x) = -\operatorname{sech}(x)\tanh(x)$ $\frac{d}{dx}\operatorname{csch}(x) = -\operatorname{csch}(x)\coth(x)$

Integration

Ex $y = \tanh^{-1}(x)$ $\frac{dy}{dx} = ?$

$\tanh(y) = x$

Implicit differentiation

$$\frac{d}{dx}(\tanh(y)) = \frac{d}{dx} x$$

chain rule ?

$$\operatorname{sech}^2(y) \cdot \frac{d}{dx} y = 1$$

$$\frac{dy}{dx} = \frac{1}{\operatorname{sech}^2(y)}$$

$$= \frac{1}{1-\tanh^2(y)} = \frac{1}{1-x^2}$$

Algebra

$$\cosh^2(t) - \sinh^2(t) = 1$$

$$\frac{1}{\cosh^2(t)}\Big(\qquad \Big)$$

$$1 - \tanh^2(t) = \operatorname{sech}^2(t)$$

Derivative formula $\Rightarrow$ anti-derivative

$$\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2} \Rightarrow \int \frac{1}{1-x^2}\,dx = \tanh^{-1}(x) + C$$

Plus 5 <u>more</u> See section Summary.