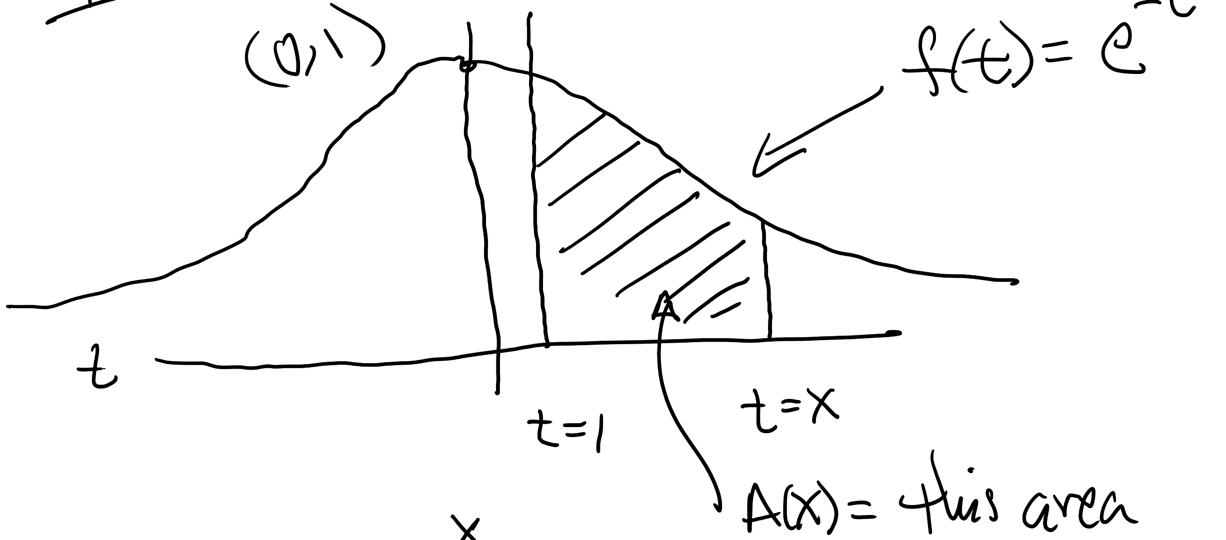
Math 181 Monday, January 25 Sections 5.4-5.5

Tue- 5.6

C/NC

Ex Define a function, $A(x)$



$$A(x) = \int_1^x e^{-t^2}\,dt$$

$$A(2) = \int_1^2 e^{-t^2}\,dt = 0.13525$$

$$A(10) = \int_1^{10} e^{-t^2}\,dt = 0.13940$$

$$A(1.2) = \int_1^{1.2} e^{-t^2}\,dt = 0.05992$$

$$A(1) = \int_1^1 e^{-t^2}\,dt = 0 \text{ (exact)}$$

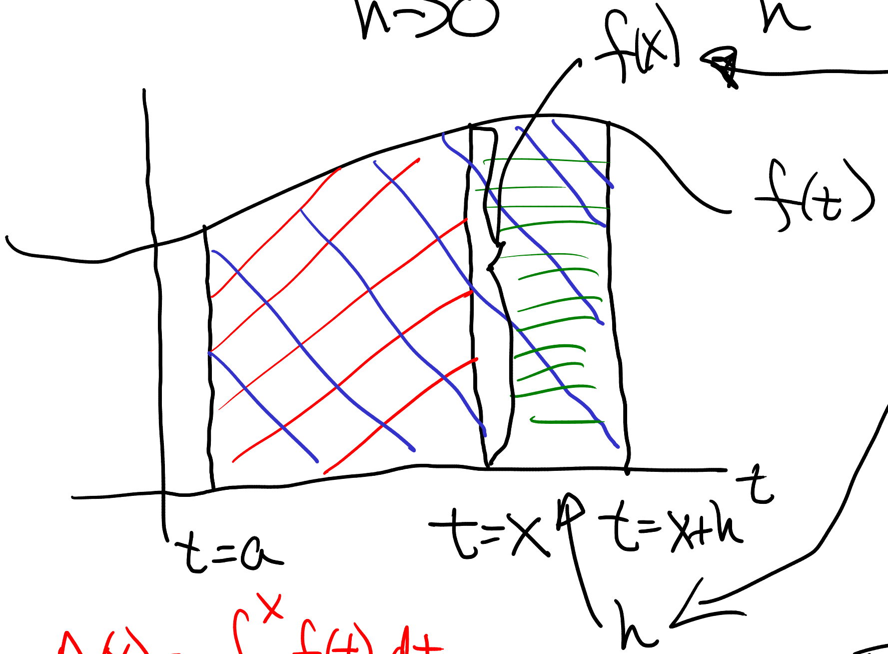
$$A(-0.5) = \int_1^{-0.5} e^{-t^2}\,dt = -1.20810$$

What is the derivative of $A(x)$?

Define $A(x) = \int_a^x f(t)\,dt$. What is $A'(x)$?

ΔA

$$A'(x) = \lim_{h\to 0} \frac{A(x+h) - A(x)}{h} = \lim_{h\to 0} \frac{1}{h}\left(A(x+h) - A(x)\right)$$



$$\boxed{A(x+h) - A(x) = \int_x^{x+h} f(t)\,dt = f(x)\,h}$$

height

width

$$= \lim_{h\to 0} \frac{1}{h}\left(f(x)\,h\right) = \lim_{h\to 0} f(x) = f(x)$$

$A(x) = \int_a^x f(t)\,dt$

$A(x+h) = \int_a^{x+h} f(t)\,dt$

Fundamental Theorem of Calculus

$$\frac{d}{dx} \int_a^x f(t)\,dx = f(x)$$

$$\frac{d}{dx}\int_1^x e^{-t^2}\,dt = e^{-x^2}$$; $\int_1^x e^{-t^2}\,dt$ is an antiderivative of e^{-x^2}

Theorem Suppose $F(x)$ is an antiderivative of $f(x)$.

Then $$\int_a^b f(x)\,dx = F(b) - F(a)$$

Proof

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$ Antidifferentiate both sides

$$\int_a^x f(t)\,dt = F(x) + C = F(x) - F(a)$$

Let $x=a$

$$\int_a^a f(t)\,dt = F(a) + C$$

$$0 = F(a) + C$$

$$C = -F(a)$$

Now $x=b$

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

P 328 "Conceptual Insight"

$$f(x) \xrightarrow{\text{integrate}} \int_a^x f(t)\,dt \xrightarrow{\text{Differentiate}} \frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$f(x) \xrightarrow{\text{differentiate}} f'(x) \xrightarrow{\text{Integrate}} \int_a^x f'(t)\,dt = f(x) - f(a)$$