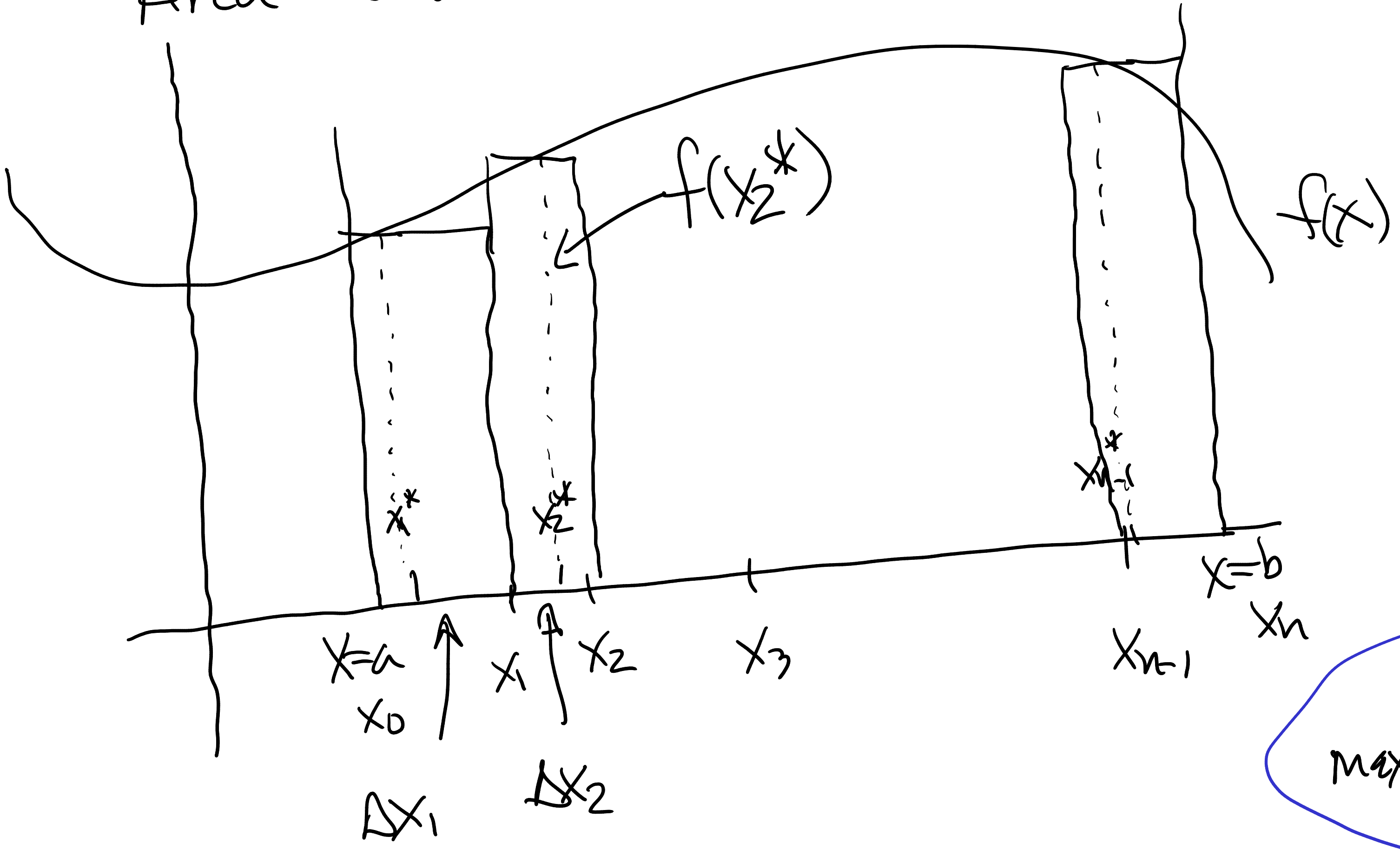
Math 181 Thursday, January 21 Section 5.2

Riemann Sums

Area under $f(x)$, $a \le x \le b$



Fri 5.3

BYOB Cartoons/Comics Anime/Magna

Mon 5.4/5.5

Typical rectangle area

$$f(x_i^*)\,\Delta x_i$$

Summation

$$\sum_i f(x_i^*)\,\Delta x_i$$

$$\lim_{\max \Delta x_i \to 0} \left(\sum_i f(x_i^*)\,\Delta x_i \right) = \int_a^b f(x)\,dx$$

Ex $\int_1^3 x^3\,dx$ $\boxed{n}$ rectangles, equal width $\Delta x = \frac{3-1}{n} = 2/n$

Partition: $x_0 = 1$, $x_1 = 1+2/n$, $x_2 = 1+2(2/n)$, $x_3 = 1+3(2/n)$, $\dots$, $x_n = 1+n(2/n) = 3$

Right hand endpoints

Evaluate x^3 $\quad (1+2/n)^3, \quad (1+2(2/n))^3, \quad \dots \quad 3^3$

$$R_n = (1+2/n)^3\, 2/n + (1+2(2/n))^3(2/n) + ((1+3(2/n))^3(2/n) + \dots + (1+n(2/n))^3(2/n)$$

$$= \sum_{i=1}^{n} \left(1+i(2/n)\right)^3(2/n) = (2/n)\sum_{i=1}^{n} 1^3 + 3(1)^2 i(2/n) + 3(1)[i(2/n)]^2 + [i(2/n)]^3$$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$$= 2/n\left[\sum_{i=1}^{n} 1 + \sum_{i=1}^{n} \frac{6}{n} i + \sum_{i=1}^{n} \frac{12}{n^2} i^2 + \sum_{i=1}^{n} \frac{8}{n^3} i^3\right] = \frac{2}{n}\left[\sum_{i=1}^{n} 1 + \frac{6}{n}\sum_{i=1}^{n} i + \frac{12}{n^2}\sum_{i=1}^{n} i^2 + \frac{8}{n^3}\sum_{i=1}^{n} i^3\right]$$

$$= \frac{2}{n}\left[n + \frac{6}{n}\,\frac{n(n+1)}{2} + \frac{12}{n^2}\,\frac{n(n+1)(2n+1)}{6} + \frac{8}{n^3}\,\frac{n^2(n+1)^2}{4}\right]$$

$$= \frac{2n}{n} + \frac{6(n(n+1))}{n^2} + \frac{4(n(n+1)(2n+1))}{n^3} + \frac{4n^2(n+1)^2}{n^4}$$

$$= 2 + (6+\dots) + (8+\dots) + (4+\dots)$$

$\frac{2n^2}{n^3}$

Now

$$\lim_{n\to\infty} R_n = 2+6+0+8+0+4+0 = 20$$