Class—FCLA SD

Advanced Linear Algebra

Robert Beezer

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From Example TIS, and Example TGE,

$$A = \begin{bmatrix} -8 & 6 & -15 & 9 \\ -8 & 14 & -10 & 18 \\ 1 & 1 & 3 & 0 \\ 3 & -8 & 2 & -11 \end{bmatrix}.$$

```
A = matrix(QQ, [
[-8,  6, -15,   9],
[-8, 14, -10,  18],
[ 1,  1,   3,   0],
[ 3, -8,   2, -11]
])
A
```

Demonstration 1 Duplicate Example TGE and build the generalized eigenspaces of $\lambda = -2$ and $\lambda = 1$. Build as Sage `.right_kernel()` with a "pivot" basis.

Demonstration 2 Grab the first basis vector of each generalized eigenspace, and check that is *not* a traditional eigenvector. Name these `x2` and `x4`. (We'll see why.)

Define $\vec{x}_1 = (A - \lambda I_4)\vec{x}_2$ and $\vec{x}_3 = (A - \lambda I_4)\vec{x}_4$ (which eigenvalues to be decided at class time). Two interesting things happen, which we illustrate mathematically with the first generalized eigenspace.

First

$$\begin{aligned} \vec{0} &= (A - \lambda I_4)^2 \vec{x}_2 \\ &= (A - \lambda I_4)(A - \lambda I_4)\vec{x}_2 \\ &= (A - \lambda I_4)\vec{x}_1. \end{aligned}$$

Which says that $\vec{x}_1$ is a traditional eigenvector of A, so $A\vec{x}_1$ is a scalar multiple of $\vec{x}_1$.

And

$$\begin{aligned} A\vec{x}_2 &= (A - \lambda I_4)\vec{x}_2 + \lambda\vec{x}_2 \\ &= \vec{x}_1 + \lambda\vec{x}_2 \end{aligned}$$

Which says that $A\vec{x}_2$ is *almost, almost, almost* a scalar multiple of $\vec{x}_2$.

Demonstration 3 Make these four vectors the columns of a matrix S, and verify that the matrix is nonsingular, and hence the columns are a basis of $\mathbb{C}^4$.

Demonstration 4 Explore similarity with A and S.