Eigenvalues are hard. Everything else is easy.
Following is verbatim from an FCLA/290 in-class worksheet.
A totally random matrix is unlikely to have a characteristic polynomial that factors if we restrict ourselves to the rationals. But we can find all the roots over $\overline{\mathbb{Q}}$, the set of all algebraic numbers. (This is the set of all real roots of all possible polynomials with integer coefficients.)

```python
D = random_matrix(QQ, 10)
p = D.characteristic_polynomial()
p.factor()
```

If we make a “block diagonal” matrix, then the characteristic polynomial will definitely factor some

```python
E = block_diagonal_matrix([random_matrix(QQ, 5), random_matrix(QQ, 5)])
p = E.charpoly()
p.factor()
```

Finally a large example, illustrating how fast Sage is at making characteristic polynomials and at factoring.

```python
F = block_diagonal_matrix([random_matrix(QQ, 50), random_matrix(QQ, 50)])
p = F.charpoly()
p.factor()
```

This is such a common operation, that Sage has a shorthand method for the Factored Characteristic Polynomial, namely .fcp().

```python
F.fcp()
```

Nothing short of amazing!