Math 491, Monday, April 27

Tue - Sage

Thu - 23/ Quintic

Fri - Problem session
    Sage 23

Mon - Exam 4 22/23

Tue - Early Start (8 AM)
    3x Presentations
    20 min + 5 min questions

Final Tuesday 10AM Pacific
Fixed field $F$, $G = \text{group of automorphisms of } F$ (not all)

$$F_G = \{ f \in F \mid \sigma(f) = f \text{ for all } \sigma \in G \}$$

Fact $F_G$ subfield of $F$ ($F$ extension of $F_G$)

Theorem $E$ is the field of splitting $F$ for a separable polynomial. Then $E \text{ is splitting field of } F$.

Theorem $F = E_G$ for some group of automorphisms of $E$

$$\Rightarrow [E : F] = |G|$$

Definition An extension $E/F$ is normal if whenever an irreducible polynomial over $F$ has a root in $E$, then all of its roots are in $E$. 

Inverse process
Extension $\mathbb{Q}(2^{1/4}) / \mathbb{Q}$.

Irreducible polynomial $x^4 - 2$.

$\mathbb{Q}(2^{1/4})$ has two roots of $x^4 - 2$, $(\pm \sqrt[4]{2})$, but does not contain two other roots ($\pm i \sqrt[4]{2}$).

So $\mathbb{Q}(2^{1/4}) / \mathbb{Q}$ is not a normal extension.

The splitting field of $x^4 - 2$ is $\mathbb{Q}(2^{1/4}, i 2^{1/4})$.

The Galois group $\sigma : i 2^{1/4} \rightarrow i 2^{1/4}$, $-i 2^{1/4} \rightarrow -i 2^{1/4}$, $2^{1/4} \rightarrow -2^{1/4}$, $-2^{1/4} \rightarrow 2^{1/4}$.

$\sigma$ has order 2.

Full Galois group $\cong D_4$.

$\langle \sigma \rangle$ not normal in $D_4$. 
Theorem. The following are equivalent for extension $E/F$.

1) $E$ finite normal separable extension.

2) $E$ splitting field over $F$ of a separable polynomial.

3) $F = E_G$ for some group $G$ of automorphisms of $E$. 
Fundamental Theorem of Galois Theory

F finite field or has characteristic zero. E finite normal extension

E \rightarrow G(E/E) \text{ trivial}

\Rightarrow \text{bijection from fields (subfields) to subgroups}

K \text{ normal extension of } F

\Leftrightarrow G(E/K) \text{ is a normal subgroup of } G(E/F)

If so, \[ G(K/F) \cong \frac{G(E/F)}{G(E/K)} \]

F \text{ infinite}

\text{inclusion-reversing}