Math 491, Monday, April 20 Chapter 23

Tue | 23
Thu |
Fri | ↕ Sage

Mon | 23
Tue |

Thu Problem Session
Fri Exam 4

Mon | Projects
Tue |

Mon ↓ Final

- Email Read-only
- Project Discussion

Defn $\alpha, \beta \in E$, E/F, are conjugate if α & β have the same minimal polynomial $\in F[x]$.

Theorem If a & b are conjugate in E/F, then there is an isomorphism $\phi: F(a) \to F(b)$ where 1) ϕ fixes F element-wise.

2) $\phi(a) = b$

"conjugate elements are indistinguishable"

Ex $\mathbb{Q}(i) / \mathbb{Q}$ $(i = \sqrt{-1}$, i root of $x^2+1)$

E F

$2+3i$ & $2-3i$ are conjugate

$\alpha = 2+3i \rightarrow \alpha - 2 = 3i \rightarrow \alpha^2 - 4\alpha + 4 = -9 \rightarrow \alpha^2 - 4\alpha + 13 = 0$

minimal polynomial $x^2 - 4x + 13$ same

$\beta = 2-3i \rightarrow \beta - 2 = -3i \rightarrow \beta^2 - 4\beta + 4 = -9 \rightarrow \beta^2 - 4\beta + 13 = 0$

minimal polynomial $x^2 - 4x + 13$

α & β are both roots of $x^2 - 4x + 13$. So α & β are conjugate.

Base-field-fixing isomorphism, $\mathbb{Q}(2+3i) \rightarrow \mathbb{Q}(2-3i)$, $2+3i \rightarrow 2-3i$?

$\rho : \mathbb{Q}(2+3i) \rightarrow \mathbb{Q}(2-3i)$ $\rho(a+bi) = a - bi$ ("complex conjugation")

$\rho((a+bi)(c+di)) = \rho((ac-bd) + (ad+bc)i) = (ac-bd) - (ad+bc)i$ / operation preserving

$= (a-bi)(c-di) = \rho(a+bi)\,\rho(c+di)$

$\overline{zw} = \overline{z}\,\overline{w}$

Fixed Fields

Defn Let G be a subgroup of $\mathrm{Aut}(F)$. Then

$$F_G = \{a \in F \mid \sigma(a) = a \text{ for all } \sigma \in G\} \subseteq F$$

F_G is the fixed field of G.

Theorem T is any subset of $\mathrm{Aut}(F)$, then

$\{a \in F \mid \sigma(a) = a$ for all $\sigma \in T\}$ is a subfield of F.

Proof Multiplicative closure set $\nearrow K$ ~~Grab~~ $a, b \in K$. Is $ab \in K$?

Take $\sigma \in T$ then $\sigma(ab) = \sigma(a)\sigma(b) = ab$ for all $\sigma \in T$

σ automorphism

<u>Theorem</u> Suppose E is the splitting field of a seperable polynomial over F. Then $E_{G(E/F)} = F$

① $E/F \Rightarrow$ Galois group $G(E/F)$, "base field fixing automorphisms"

② $E_{G(E/F)} \Rightarrow$ subfield of E fixed element-wise by a set of automorphisms

<u>At least</u> F. "Inverse processes"