Math 491, Monday, April 6 Problem Session, Chapter 21

Tue - Exam Chap 20/21 (not Invariant Subspaces)

Thu - Chapter 22, RQ 6AM Pacific

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21 3b Splitting field of x^4+1 over $\mathbb{Q}$

$\mathbb{Q}(i,\sqrt{2})$ $\quad x^4 = -1 \quad (x^2)^2 = -1 \quad x^2 = \pm\sqrt{-1} = \pm i$

$x = \pm\sqrt{\pm i}$

$\mathbb{Q}(i)$ $\quad (x^2 - i)(x^2 + i)$

$\alpha = e^{i\pi/2}$

$\mathbb{Q}$ $\quad \alpha^{1/2} = (e^{i\pi/2})^{1/2} = e^{i\pi/4} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$\alpha = -i = e^{i\,3\pi/4}$

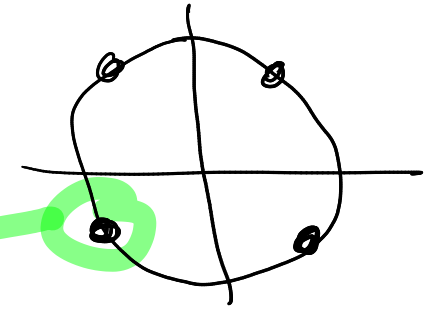
$\alpha^{1/2} = e^{i\,3\pi/8}$

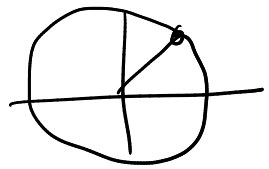
$\alpha = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

$$= \left[x - \left(\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\right)\right]\left[x + \left(\tfrac{\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\right)\right]\left[x - \left(\tfrac{-\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\right)\right]\left[x + \left(\tfrac{-\sqrt{2}}{2} + \tfrac{\sqrt{2}}{2}i\right)\right]$$

"split"

$\mathbb{Q}(i,\sqrt{2})$ $\quad [\mathbb{Q}(i,\sqrt{2}) : \mathbb{Q}] = 4$





16/ $\text{char}(F) = p$ $\quad p(x) = x^p - a \quad \Rightarrow$ irreducible over F or splits over F

Irreducible $\Rightarrow$ done

Reducible $\quad p(x) = r(x)\,s(x)$ $\quad r(x)$ irreducible, β root of $r(x)$

E extension of F w/ a root of $p(x)$ $\quad E \supseteq F$, $a \in F$

E

$p \left[\begin{array}{l} F(\beta) \\ F \end{array} \right]$ degree less than p, degree divides p

$[E:F] = [E:F(\beta)][F(\beta):F]$

$p = [E:F(\beta)]$ (less than p)

$\Rightarrow [F(\beta):F] = 1 \Rightarrow F(\beta) = F \Rightarrow \beta \in F$

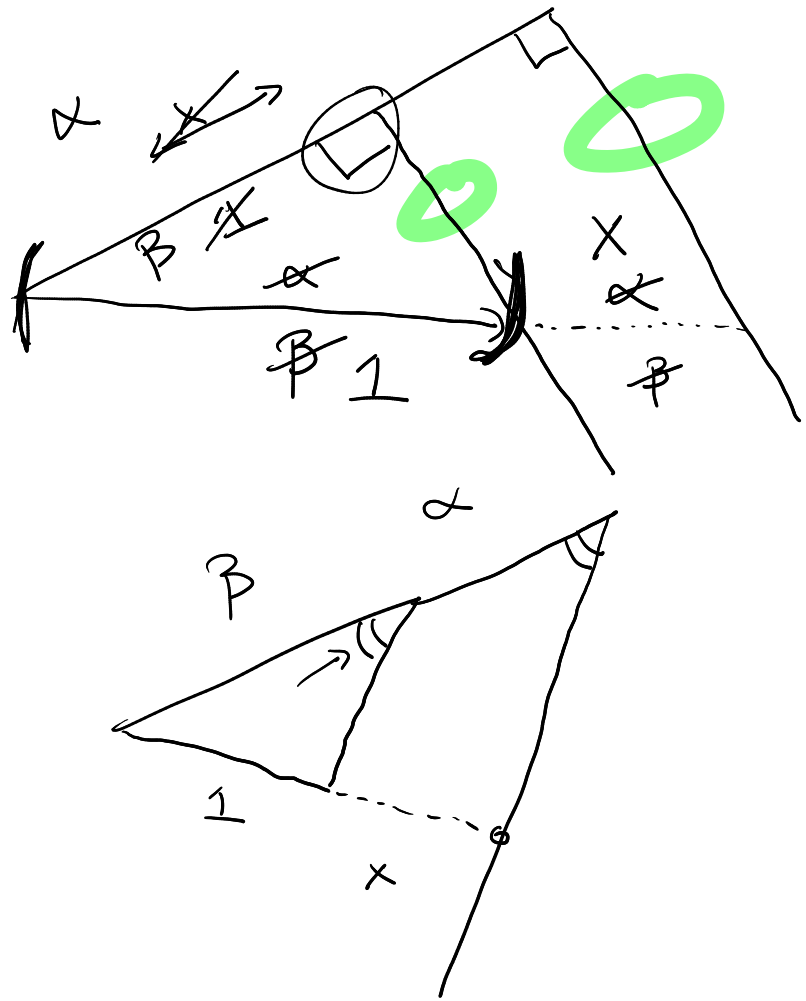
$0 = p(\beta) = \beta^p - a \Rightarrow \beta^p = a$

$x^p - a = x^p - \beta^p = (x-\beta)^p$

Freshman's Dream (char $F = 0$)

linear factors, $\beta \in F$ in $F[x]$

23.19 $\beta \Leftrightarrow x$ $\alpha \Leftrightarrow \beta$ Givens: 1, α, β



$$\frac{\alpha}{\beta} = \frac{x}{1}$$

β

$\beta < 1$