Math 491, Friday, April 3 Chapter 21 Fields

Sun 11:59 Project Proposal

Mon Problem Session
Sage 21

Tue Exam 3 Chaps 20/21
8:30

Thu Chapter 22 Finite Fields
RQ 6AM

Algebraic numbers: all roots of all polys in $\mathbb{Q}[x]$ (or $\mathbb{Z}[x]$)

$\overline{\mathbb{Q}}$ (Sage: QQbar)

"Algebraic reals" $\overline{\mathbb{Q}} \cap \mathbb{R}$ (Sage: AA)

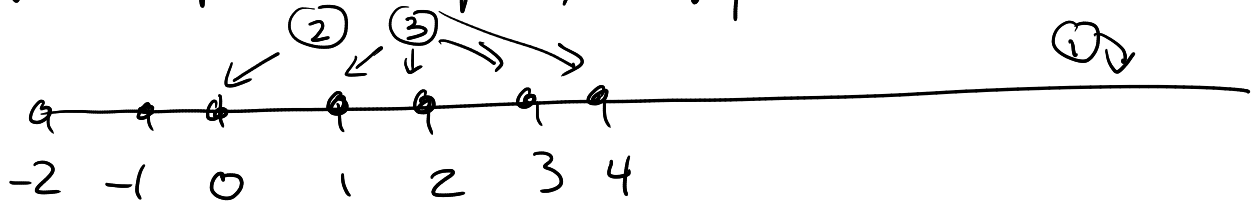
Straightedge & Compass Constructions

"Constructible numbers" ⊆ set of points in $\mathbb{R}^2$

Compass: "collapsable" "fixed"

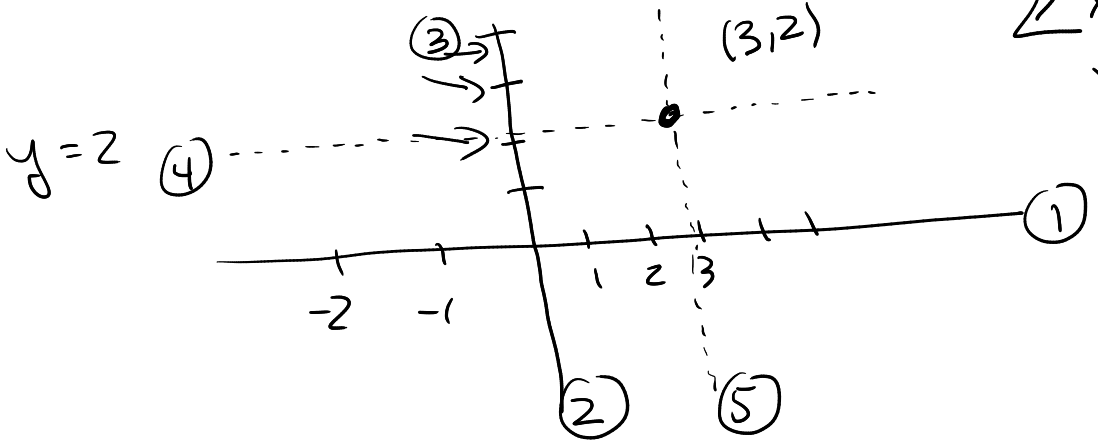
Start w/ unit length 

Non-collapsible compass, copy distances

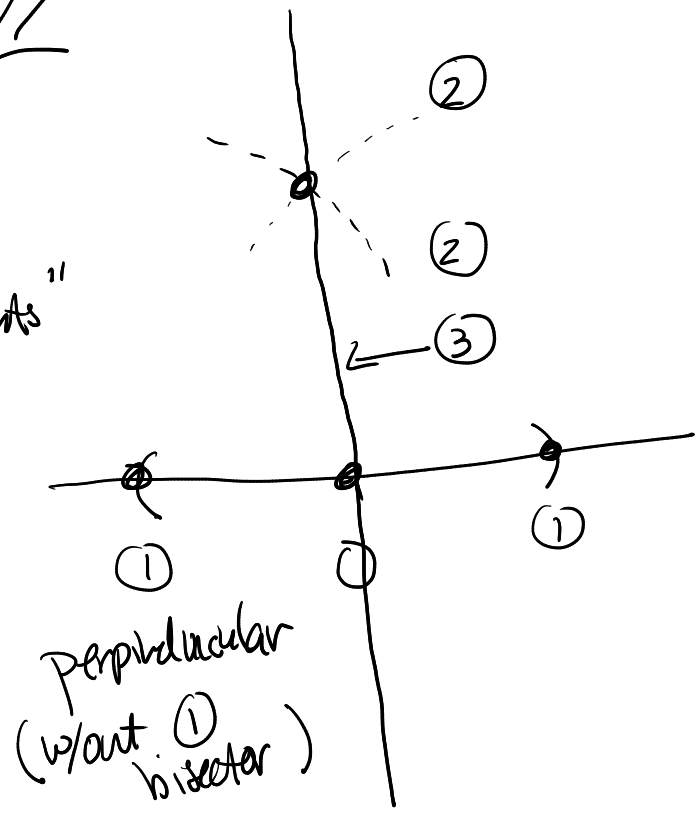


$\mathbb{Z}$

④ Perpendicular at 0



$\mathbb{Z} \times \mathbb{Z}$ "lattice points"

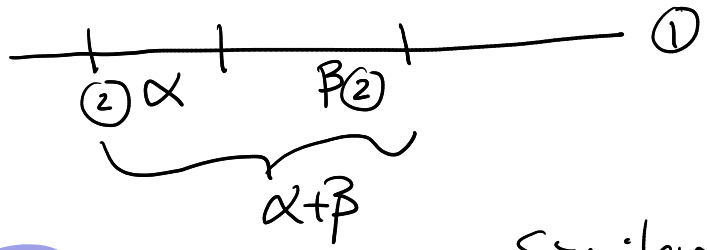


perpendicular (w/out ① bisector)

Defn Constructible numbers is set of all lengths that can be constructed w/ straight edge & compass.

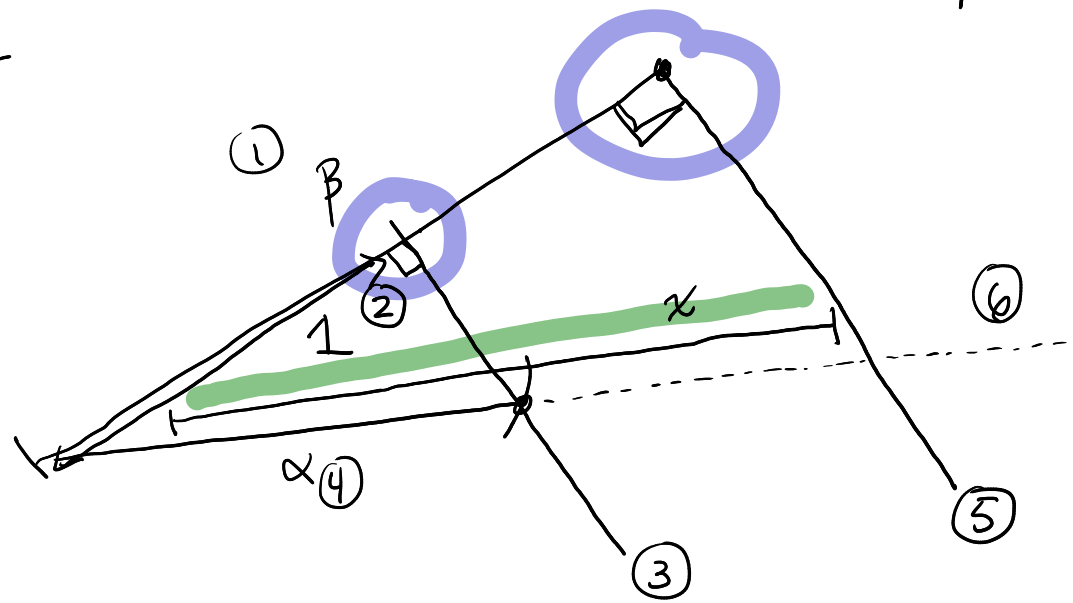
Fact Constructible numbers is a field.

Proof closure $\alpha+\beta$



$\alpha, \beta \in C \Rightarrow \alpha\beta \in C$

$\beta > 1$



Similar triangles

$$\frac{x}{\alpha} = \frac{\beta}{1}$$

$$x = \alpha\beta$$

"Division" (units) similar

2-D Start w/ $\mathbb{Z} \times \mathbb{Z}$

Build linear systems

$2x + 3y = 9$ ← draw these lines

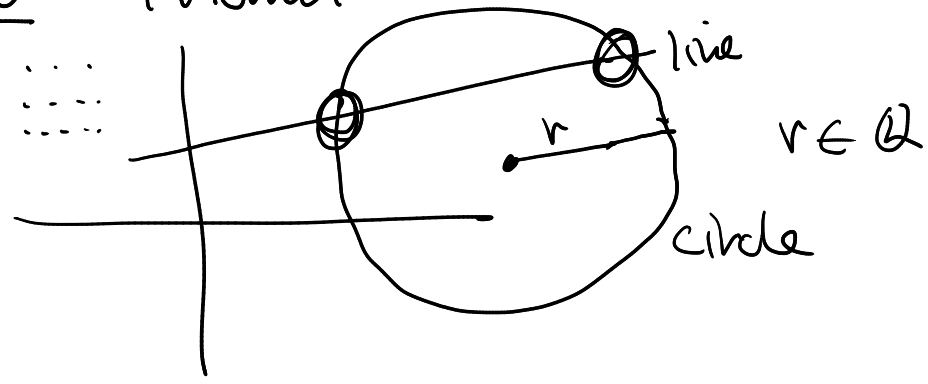
$-x + 6y = 17$ ←

solution is in $\mathbb{Q} \times \mathbb{Q}$

solution is intersection

$\mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Q} \times \mathbb{Q}$

Also intersect a line w/ a circle



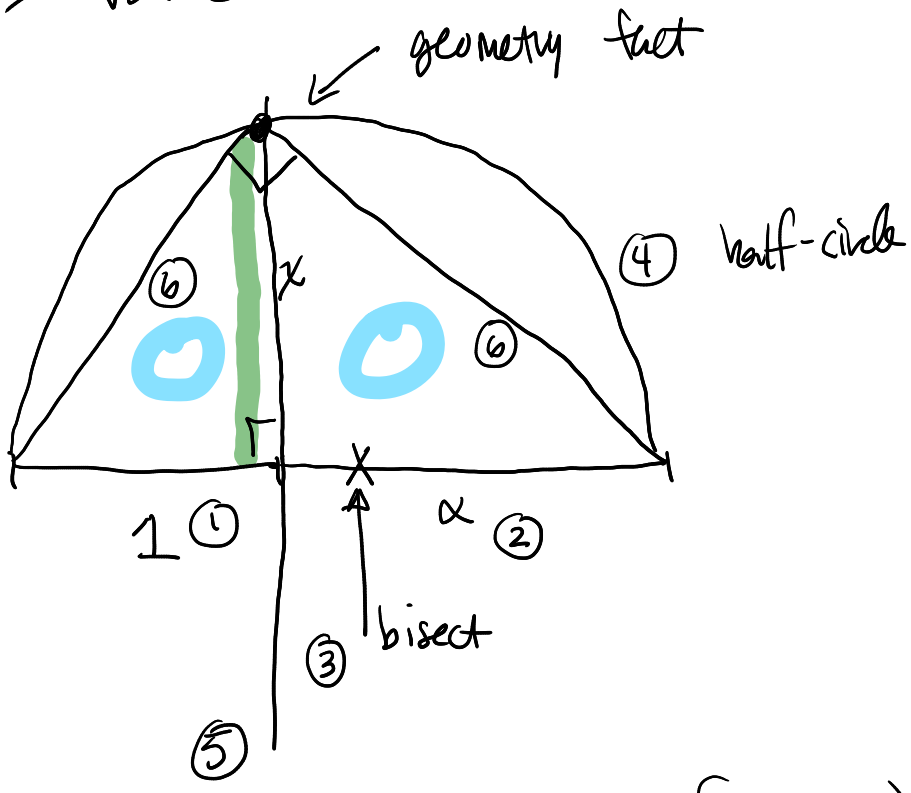
$r \in \mathbb{Q}$

$\mathbb{Q} \times \mathbb{Q}$

Solve system $\begin{cases} ax + by + c = 0 \\ dx^2 + ex + fy^2 + gy + h = 0 \end{cases}$

produce square roots

$\alpha \in C \Rightarrow \sqrt{\alpha} \in C \qquad \alpha > 1$



Similar triangles

$$\frac{x}{1} = \frac{\alpha}{x}$$

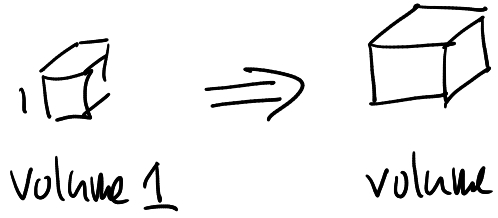
$$x^2 = \alpha$$

$$x = \sqrt{\alpha}$$

$$2\left[\begin{matrix} Q_k \\ \vdots \\ Q_2 \\ 1 \\ Q_1 \\ 1 \\ Q_0 \end{matrix}\right.$$

Fact If α is constructible $[\mathbb{Q}(\alpha) : \mathbb{Q}] = 2^k$

Double the cube

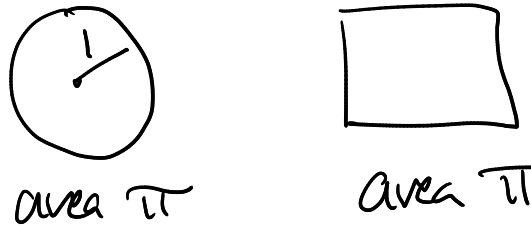


requires side length $\sqrt[3]{2}$

$[\mathbb{Q}(\sqrt[3]{2}) : \mathbb{Q}] = 3$ ← not a power of 2

So $\sqrt[3]{2}$ not constructible

Square the circle



requires side length $\sqrt{\pi}$

$[\mathbb{Q}(\sqrt{\pi}) : \mathbb{Q}] = ?$

π transcendental

not a finite extension

Trisect an Angle

60° can't be trisected

$\cos(20^\circ)$ is not constructible.

$\cos(20^\circ)$ root of irreducible

$x^3 - \frac{3}{4}x - \frac{1}{8}$ (triple angle identities)

$[\mathbb{Q}(\cos(20^\circ)) : \mathbb{Q}] = 3$ ← not a power of 2

Sage:

not(log(2, QQ[cos(pi/9)]) in ZZ)

True