Math 290 B, Monday, May 4, Section CB

Mon- CB

Tue - Problem Session
House keeping
Writing (11AM)

Wed- Exam R

Tue, May 12 Final Exam
9-12 Pacific
(8-11 by appointment)

Change-of-Basis

matrix representation of identity L.T.

$I_V : V \to V \qquad I_V(\underset{\sim}{v}) = \underset{\sim}{v}$

B, C bases of V

$$M^{I_V}_{B,C} = C_{B,C}$$

__Fact__ $\rho_C(I_V(\underset{\sim}{v})) = M^{I_V}_{B,C}\, \rho_B(\underset{\sim}{v})$ FTMR

$$\underline{\rho_C(\underset{\sim}{v})} = C_{B,C}\, \underline{\rho_B(\underset{\sim}{v})}$$

Ex M_{22} $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

$C = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} \right\}$

$$C_{C,B} = \begin{bmatrix} 1 & 2 & 1 & 2 \\ -2 & -3 & -3 & -2 \\ 1 & 3 & 1 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix} \qquad C_{B,C} = C_{C,B}^{-1} = \begin{bmatrix} \quad \end{bmatrix}^{-1} = \begin{bmatrix} 6 & -1 & -5 & 2 \\ -1 & 2 & 3 & -2 \\ -3 & -1 & 1 & 0 \\ 0 & -1 & -1 & 1 \end{bmatrix}$$

Catch-up MR: T invertible $\quad M^{T^{-1}}_{C,B} = \left(M^{T}_{B,C}\right)^{-1}$ (see MR. C40)

$T: U \to V$ $\quad B, C$ bases U; D, E bases of V

$$\boxed{?\; M^{T}_{DB} = C_{DE}\, M^{T}_{EC}\, C_{CB}}$$

$$M^{T}_{B,D} = C_{E,D}\, M^{T}_{C,E}\, C_{B,C}$$

$$\rho_D(T(\underset{\sim}{u})) \leftarrow \rho_B(\underset{\sim}{u}) \quad \| \quad \rho_D(T(\underset{\sim}{u})) \leftarrow \rho_E(T(\underset{\sim}{u})) \leftarrow \rho_C(\underset{\sim}{u}) \leftarrow \rho_B(\underset{\sim}{u})$$

Ex $D = \{1, x, x^2\}$, $E = \{1, 1+x, 1+x+x^2\}$

$$\underset{3\times 4}{\begin{bmatrix} -5 & -12 & -5 & -12 \\ 10 & 20 & 12 & 16 \\ 0 & 2 & -1 & 4 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}} \underset{3\times 4}{\begin{bmatrix} 2 & 1 & 3 & -2 \\ 5 & 3 & 7 & -4 \\ 1 & 1 & 1 & 0 \end{bmatrix}} \underset{4\times 4}{\begin{bmatrix} 1 & 2 & 1 & 2 \\ -2 & -3 & -3 & -2 \\ 1 & 3 & 1 & 4 \\ -1 & 0 & -2 & 3 \end{bmatrix}}$$

$$M^T_{C,E} \qquad C_{D,E} \qquad M^T_{B,D} \qquad C_{C,B}$$

$$(C^{-1}_{E,D})$$

Specialize $T : U \to U$ $\quad B = D \quad C = E$

$$M^T_{B,B} = C_{C,B}\, M^T_{C,C}\, C_{B,C}$$

$$= C^{-1}_{B,C}\, M^T_{C,C}\, C_{B,C}$$

similarity $\underline{\underline{\text{is}}}$ change-of-basis

<u>Defn</u> $T: V \to V$ then $\underset{\sim}{v} \in V$ is an <u>eigenvector</u> of T if $T(\underset{\sim}{v}) = \lambda \underset{\sim}{v}$

<u>Diagonalization</u>

1. Build an <u>easy</u> matrix representation (nice basis B)
2. Compute eigenvalues, eigenvectors (column vectors)
3. Un coordinatize relative to B (ρ_B^{-1})
4. New basis, C, for V w/ eigenvectors (in V)
5. Build a matrix representation of T relative to C

(See first MR in notes.)