Math 290B, Monday, April 13 Section SD

Mon - SD
Sage

Tue - Problem Session
Bring Your Pet to Class Day
Writing D&E-2-beezer.pdf

Wed - Exam D&E

Thu - LT (RQ!)

Fri - ILT

Sage determinant
.fcp()
RREF

Similarity A & B similar if there exists S so that $S^{-1}AS = B$

Equivalence Relation

Theorem If A & B are similar then A & B have same eigenvalues.

Proof There is an S so that $S^{-1}AS = B$.

$$P_B(\lambda) = \det(B - \lambda I) = \det(S^{-1}AS - \lambda I) = \det(S^{-1}AS - \lambda S^{-1}S)$$

$$= \det(S^{-1}(A - \lambda I)S) = \det(S^{-1})\det(A - \lambda I)\det(S)$$

$$= \det(S^{-1})\det(S)\det(A - \lambda I) = \det(S^{-1}S)\,P_A(\lambda)$$

$$= \det(I)\,P_A(\lambda) = 1\,P_A(\lambda) = P_A(\lambda)$$

Eigenvalues of A & B (roots of characteristic polys) are "the same".

NOT THE CONVERSE

Ex $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ eigenvalues: $\lambda = 1, 1$

Are A & B similar? NO $S^{-1}AS = S^{-1}I_2S = S^{-1}S = I_2 \neq B$

Diagonalization Given A, is there a matrix S and a diagonal matrix D so that $S^{-1}AS = D$?

Sometimes If so D has eigenvalues on diagonal & S has eigenvectors for columns

Theorem DC

$A_{n \times n}$ diagonalizable $\iff$ there is a set of n linearly independent eigenvectors for A.

Proof ($\Leftarrow$) Assume $T = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_n\}$ is set of linearly independent vectors for eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. Define

$$D = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \; \lambda_n \end{bmatrix} \qquad S = [\underset{\sim}{x}_1 | \underset{\sim}{x}_2 | \cdots | \underset{\sim}{x}_n] \text{ nonsingular}$$

$$= [\lambda_1 \underset{\sim}{e}_1 | \lambda_2 \underset{\sim}{e}_2 | \ldots | \lambda_n \underset{\sim}{e}_n]$$

$$AS = A[\underset{\sim}{x}_1 | \underset{\sim}{x}_2 | \ldots | \underset{\sim}{x}_n] = [A\underset{\sim}{x}_1 | A\underset{\sim}{x}_2 | \ldots | A\underset{\sim}{x}_n] = [\lambda_1 \underset{\sim}{x}_1 | \lambda_2 \underset{\sim}{x}_2 | \ldots | \lambda_n \underset{\sim}{x}_n]$$

$$= [\lambda_1 S\underset{\sim}{e}_1 | \lambda_2 S\underset{\sim}{e}_2 | \ldots \lambda_n S\underset{\sim}{e}_n] = [S(\lambda_1 \underset{\sim}{e}_1) | S(\lambda_2 \underset{\sim}{e}_2) | \ldots | S(\lambda_n \underset{\sim}{e}_n)]$$

$$= S[\lambda_1 \underset{\sim}{e}_1 | \lambda_2 \underset{\sim}{e}_2 | \ldots | \lambda_n \underset{\sim}{e}_n] = SD$$

$$AS = SD \rightarrow S^{-1}AS = D$$