Math 290 B, Monday, April 6, Problem Session, Chapter VS

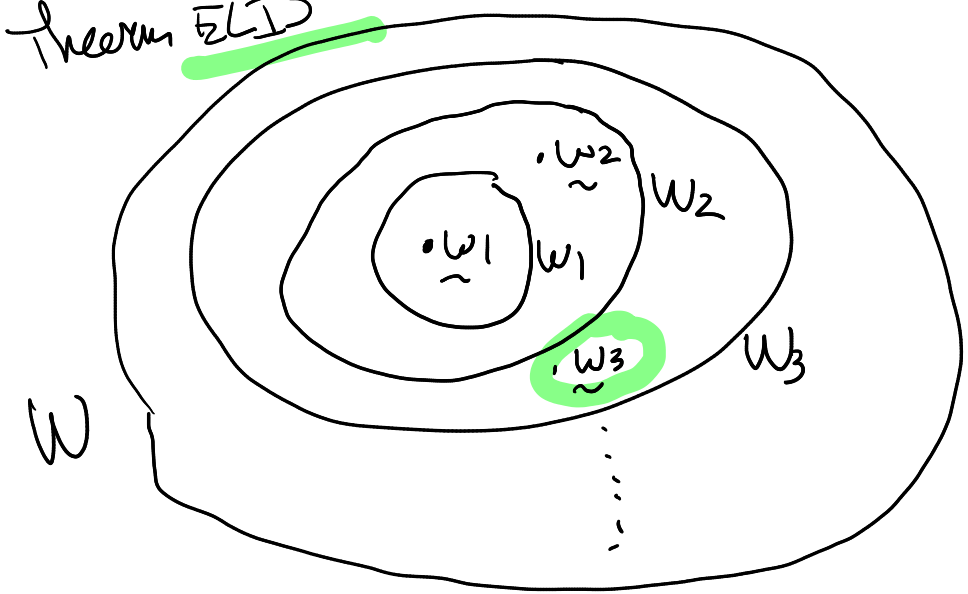
Tue - Section PDM
Sage

Wed - Exam VS

Thu - Section EE

PD. T25 $\dim(V)$ finite, W any subspace $\Rightarrow$ $\dim(W)$ finite

Theorem ELIS



$w_1 \neq 0$, $w_1 \in W$, define $W_1 = \langle \{w_1\} \rangle$, $\{w_1\}$ l.i.

$w_2 \in W_1$, $w_2 \in W$, define $W_2 = \langle \{w_1, w_2\} \rangle$, $\{w_1, w_2\}$ l.i.

⋮

Process stops with a l.i. set, that spans W

Stops before the set has a greater size than a basis for V

B.M20 Example BM Basis for M_{mn}

M_{23} $B = \left\{ \begin{bmatrix} 1&0&0\\0&0&0 \end{bmatrix}, \begin{bmatrix} 0&1&0\\0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&1\\0&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\1&0&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\0&1&0 \end{bmatrix}, \begin{bmatrix} 0&0&0\\0&0&1 \end{bmatrix} \right\}$

spans? $\begin{bmatrix} 2&-3&4\\9&11&-6 \end{bmatrix} = 2\begin{bmatrix} 1&0&0\\0&0&0 \end{bmatrix} + (-3)\begin{bmatrix} & & \\ & & \end{bmatrix} + \cdots + (-6)\begin{bmatrix} 0&0&0\\0&0&1 \end{bmatrix}$

$\in M_{23}$

l.i.? $\begin{bmatrix} 0&0&0\\0&0&0 \end{bmatrix} = \underset{\sim}{0} = \alpha_1\begin{bmatrix} 1&0&0\\0&0&0 \end{bmatrix} + \alpha_2\begin{bmatrix} & & \\ & & \end{bmatrix} + \cdots + \alpha_6\begin{bmatrix} 0&0&0\\0&0&1 \end{bmatrix}$

$= \begin{bmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \alpha_4 & \alpha_5 & \alpha_6 \end{bmatrix} \Rightarrow \alpha_1=0\ \alpha_2=0\ \alpha_3=0\ \alpha_4=0\ \alpha_5=0\ \alpha_6=0$

$B_{k,l}$ = 1 in entry k,l, 0 otherwise $\quad [B_{k,l}]_{ij} = \begin{cases} 1 & \text{if } i=k, j=l \\ 0 & \text{otherwise} \end{cases}$

Span Grab $A \in M_{mn}$

Claim $A = \sum_{k,l} [A]_{k,l} B_{k,l}$

$$[A]_{ij} = \left[\sum_{k,l} [A]_{kl} B_{kl}\right]_{ij}$$

$$= \sum_{k,l} \left[[A]_{kl} B_{kl} \right]_{ij} \quad \text{Defn MA}$$

$$= \sum_{k,l} [A]_{kl} [B_{kl}]_{ij} \quad \text{when } k,l \neq i,j \text{ terms are zero}$$

$$= [A]_{ij} \, 1$$

$$= [A]_{ij} \qquad \text{So } A = \sum_{k,l} [A]_{kl} B_{kl}$$

Spring 2016 / Exam VS
P_2 #3

(c) Let S, $|S| = 4$. Does S span P_2? $\dim(P_2) = 3$ (Theorem)

$\{1, x, x^2, 1+x+x^2\}$ spans P_2

$\{1+x, 2+2x, 10+10x, 100+100x\}$ doesn't span P_2