

Math 290 B, Friday, April 3 Section DM

Mon- Problem Session (VS)
Writing VS (11:00)

Tue- PDM

Wed- Exam VS \nearrow Idet

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-	+	-	+

Ex

$$A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 5 & 0 & 0 & 3 \\ 2 & -1 & 1 & 4 \\ 0 & 3 & 1 & 2 \end{bmatrix}$$

$$\det(A) = |A| = 5(1)$$

$$= 5 \begin{vmatrix} -1 & 3 & 2 \\ -1 & 1 & 4 \\ 3 & 1 & 2 \end{vmatrix} + 0(1) \begin{vmatrix} 2 \\ 2 \\ 0 \end{vmatrix} + 0(1) \begin{vmatrix} 2 \\ 2 \\ 0 \end{vmatrix} + 3(1) \begin{vmatrix} 2 & -1 & 3 \\ 2 & -1 & 1 \\ 0 & 3 & 1 \end{vmatrix}$$

$$= 5 \left(-1 \begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} - 3 \begin{vmatrix} -1 & 4 \\ 3 & 2 \end{vmatrix} + 2 \begin{vmatrix} -1 & 1 \\ 3 & 1 \end{vmatrix} \right) + 3 \left(0 \cdot () - 3 \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} \right)$$

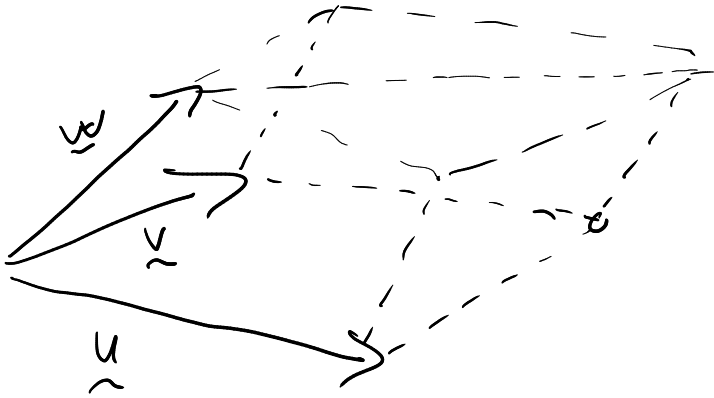
$$= 5(2 + 42 + -10) + 3(12) = -170 + 36 = -134$$

Ex

A: $\begin{bmatrix} 2 & -1 & 3 & 4 \\ 0 & 4 & -1 & 8 \\ 0 & 0 & -2 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\det(A) = 2 \cdot 4 \cdot \begin{vmatrix} -2 & 5 \\ 0 & 1 \end{vmatrix} = 2 \cdot 4 \cdot (-2 \cdot 1) = -16$$

Determinant is a function that takes square matrices to numbers



$$|(\underline{u} \times \underline{v}) \cdot \underline{w}| \text{ area of "box" volume}$$

$$= \begin{vmatrix} \underline{u} & \underline{v} & \underline{w} \end{vmatrix} = \begin{vmatrix} \sim \\ \sim \\ \sim \end{vmatrix}$$

Elementary Matrices

E_x

$$E_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & -1 & 6 \\ 1 & 0 & 4 & 2 \\ 2 & -2 & 5 & 8 \\ 3 & 1 & -1 & 3 \end{bmatrix}$$

$E_{23} \uparrow$
matrix multiplication

$$E_{23} A = \begin{bmatrix} 2 & 3 & -1 & 6 \\ 2 & -2 & 5 & 8 \\ 1 & 0 & 4 & 2 \\ 3 & 1 & -1 & 3 \end{bmatrix} = B$$

same as row swap
 $R_2 \leftrightarrow R_3$
 $\Rightarrow \det(A) = -\det(B)$

Properties of Determinants

$$\det(E_{23}) = -1$$

1) $\det(A) = \det(A^t)$

2) If A has a row of zeros (or a column of zeros)
then $\det(A) = 0$.

③ If A has two identical columns (or two identical rows) then $\det(A) = 0$

④ Suppose B is obtained from A by swapping two rows (or two columns). Then $\det(A) = -\det(B)$

⑤ Suppose B is obtained from A by multiplying a row (or a column) by α . Then $\det(B) = \alpha \det(A)$

⑥ Suppose B is obtained from A by multiplying a row (or column) and adding it to another. Then $\det(B) = \det(A)$.

row ops