Ex $W = \left\langle \left\{ \begin{bmatrix} -4 \\ -2 \\ 3 \\ -11 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} \right\} \right\rangle \subseteq \mathbb{C}^4$

Monday's Example Corrected

Basis of W? Theorem BRS, matrix w/ vectors as rows & RREF

$$\xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\} \text{ is a } \underline{\text{basis}} \text{ for } W$$

$$\underset{\sim}{w} = -2\begin{bmatrix} -4 \\ -2 \\ 3 \\ -11 \end{bmatrix} + -5\begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix} + 3\begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix} + 0\begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} = \begin{bmatrix} 34 \\ -43 \\ \boxed{36} \\ -16 \end{bmatrix} \in W$$

-36

$$\underset{\sim}{w} = 34\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + -43\begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + \boxed{36}\begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 34 \\ -43 \\ \boxed{36} \\ -16 \end{bmatrix}$$

-36 $\qquad$ -36

$\Uparrow$!

Math 290B, Tuesday, March 31 Section D

Wed - Exam M (11:00)

Thu - PD (RQ)

Fri - Problem Session

Mon - DM (RQ) / Writing VS

<u>Homework</u> (for Thursday)

$$A = \begin{bmatrix} 1 & 2 & -1 & -6 \\ 0 & 1 & -2 & -2 \\ 1 & 1 & 1 & -4 \end{bmatrix}, \quad N(A)?$$ Theorem BNS, <u>basis</u>

Form a new basis for $N(A)$, <u>different</u> from everybody elses.

Theorem $\underline{\underline{\text{SSLD}}}$ V - vector space, $S = \{\underset{\sim}{v}_1, \underset{\sim}{v}_2, \ldots, \underset{\sim}{v}_t\} \subseteq V$

S spans V $\Rightarrow$ $\underline{\text{Any}}$ set of $t+1$ or more vectors is linearly dependent.

$\underline{\text{Proof}}$ Consider $R = \{\underset{\sim}{u}_1, \underset{\sim}{u}_2, \ldots, \underset{\sim}{u}_m\} \subseteq V$ w/ $\underline{m > t}$

$$\underset{\sim}{u}_1 = a_{11}\underset{\sim}{v}_1 + a_{21}\underset{\sim}{v}_2 + \cdots + a_{t1}\underset{\sim}{v}_t$$

$$\underset{\sim}{u}_2 = a_{12}\underset{\sim}{v}_1 + a_{22}\underset{\sim}{v}_2 + \cdots + a_{t2}\underset{\sim}{v}_t$$

$$\vdots \qquad \vdots$$

$$\underset{\sim}{u}_m = a_{1m}\underset{\sim}{v}_1 + a_{2m}\underset{\sim}{v}_2 + \cdots + a_{tm}\underset{\sim}{v}_t$$

$\underline{S \text{ spans } V}$

Form system of equations

$$\boxed{a_{11}x_1 + a_{12}x_2 + \cdots + a_{1m}x_m} = 0$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2m}x_m = 0$$

$$\vdots$$

$$a_{t1}x_1 + a_{t2}x_2 + \cdots + a_{tm}x_m = 0$$

homogeneous

m variables

t equations

$\underline{\text{HMVEI}}$ $\Rightarrow$ infinitely many solutions

$R = \{\underset{\sim}{u}_j \mid 1 \le j \le m\}$

$$\underset{\sim}{u}_j = \sum_{i=1}^{t} a_{ij}\underset{\sim}{v}_i \qquad 1 \le j \le m$$

$$\sum_{j=1}^{m} a_{ij}x_j = 0 \qquad 1 \le i \le t$$

Grab a non-trivial solution

$$x_1 = c_1,\ x_2 = c_2,\ \dots,\ x_m = c_m$$

(not all c_i are zero)

Form

$$c_1 \underset{\sim}{u}_1 + c_2 \underset{\sim}{u}_2 + \cdots + c_m \underset{\sim}{u}_m$$

$$= c_1 (a_{11} \underset{\sim}{v}_1 + a_{21} \underset{\sim}{v}_2 + \cdots + a_{t1} \underset{\sim}{v}_t)$$
$$+ c_2 (a_{12} \underset{\sim}{v}_1 + a_{22} \underset{\sim}{v}_2 + \cdots + a_{t2} \underset{\sim}{v}_t)$$
$$\vdots$$
$$+ c_m (a_{1m} \underset{\sim}{v}_1 + a_{2m} \underset{\sim}{v}_2 + \cdots + a_{tm} \underset{\sim}{v}_t)$$

$$= (a_{11} c_1 + a_{12} c_2 + \cdots + a_{1m} c_m) \underset{\sim}{v}_1$$
$$+ (a_{21} c_1 + a_{22} c_2 + \cdots + a_{2m} c_m) \underset{\sim}{v}_2$$
$$\vdots$$
$$+ (a_{t1} c_1 + a_{t2} c_2 + \cdots + a_{tm} c_m) \underset{\sim}{v}_t$$

$$= 0 \underset{\sim}{v}_1 + 0 \underset{\sim}{v}_2 + \cdots + 0 \underset{\sim}{v}_t = \underset{\sim}{0} + \underset{\sim}{0} + \cdots + \underset{\sim}{0} = \underset{\sim}{0}$$

Non trivial relation of linear dependence on R.

$$x_j = c_j \quad 1 \le j \le m$$

$$\sum_{j=1}^{m} c_j \underset{\sim}{u}_j$$

$$= \sum_{j=1}^{m} c_j \left(\sum_{i=1}^{t} a_{ij} \underset{\sim}{v}_i \right)$$

$$= \sum_{j=1}^{m} \sum_{i=1}^{t} a_{ij} c_j \underset{\sim}{v}_i$$

$$= \sum_{i=1}^{t} \sum_{j=1}^{m} a_{ij} c_j \underset{\sim}{v}_i$$

$$= \sum_{i=1}^{t} \left(\sum_{j=1}^{m} a_{ij} c_j \right) \underset{\sim}{v}_i$$

$$= \sum_{i=1}^{t} 0 \underset{\sim}{v}_i = \sum_{i=1}^{t} \underset{\sim}{0} = \underset{\sim}{0}$$