Math 290 B, Monday, March 30, Section B

Mon - B (RQ)

Tue - D

Wed - Exam M (usual time 11:00)

Thu - PD

Fri - Problem session

Mon - DM / Writing VS

<u>Defn</u> V - vector space, $B = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_m}\} \subseteq V$

Then B is a <u>basis</u> for V if
1. B linearly independent
2. B spans V

Theorems BNS, BCS, BS, BRS, VRRB, ...

Ex $W = \left\langle \left\{ \begin{bmatrix} -4 \\ -2 \\ 3 \\ -11 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix}, \begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix}, \begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} \right\} \right\rangle \subseteq \mathbb{C}^4$

Basis of W? Theorem BRS, matrix w/ vectors as rows & RREF

$$\xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} \right\} \text{ is a } \underline{\text{basis}} \text{ for } W$$

$$\underset{\sim}{w} = -2 \begin{bmatrix} -4 \\ -2 \\ 3 \\ -11 \end{bmatrix} + -5 \begin{bmatrix} -2 \\ 7 \\ 3 \\ 9 \end{bmatrix} + 3 \begin{bmatrix} 6 \\ -4 \\ -7 \\ 5 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 0 \\ 3 \\ -4 \end{bmatrix} + 0 \begin{bmatrix} -4 \\ 5 \\ -5 \\ 11 \end{bmatrix} = \begin{bmatrix} 34 \\ -43 \\ 36 \\ -16 \end{bmatrix} \in W$$

$$\underset{\sim}{w} = 34 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} + -43 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 2 \end{bmatrix} + 36 \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 34 \\ -43 \\ 36 \\ \ \end{bmatrix}$$

?

34   -86 -36

Theorem VRRB V - vector space, $B = \{\underset{\sim}{v_1}, \underset{\sim}{v_2}, \ldots, \underset{\sim}{v_m}\} \subseteq V$ is a basis. $\underset{\sim}{v} \in V$.

Then $\underset{\sim}{v} = a_1 \underset{\sim}{v_1} + a_2 \underset{\sim}{v_2} + \cdots + a_m \underset{\sim}{v_m}$, uniquely.

Proof $\underset{\sim}{v} = a_1 \underset{\sim}{v_1} + a_2 \underset{\sim}{v_2} + a_3 \underset{\sim}{v_3} + \cdots + a_m \underset{\sim}{v_m}$ because B spans V.

Assume also $\underset{\sim}{v} = b_1 \underset{\sim}{v_1} + b_2 \underset{\sim}{v_2} + \cdots + b_m \underset{\sim}{v_m}$ (Proof Technique U)

Then

$$\underset{\sim}{0} = \underset{\sim}{v} - \underset{\sim}{v} = (a_1 \underset{\sim}{v_1} + \cdots + a_m \underset{\sim}{v_m}) - (b_1 \underset{\sim}{v_1} + \cdots + b_m \underset{\sim}{v_m})$$

vector operations

$$= (a_1 - b_1)\underset{\sim}{v_1} + (a_2 - b_2)\underset{\sim}{v_2} + \cdots + (a_m - b_m)\underset{\sim}{v_m}$$

RLD on a LI set

$$\Rightarrow (a_1 - b_1) = 0, \ldots (a_2 - b_2) = 0, \ldots, (a_m - b_m) = 0$$

$a_1 = b_1$ $a_2 = b_2$ $a_m = b_m$