Math 290A, Tuesday, April 28 Section MR

Wed - Exam hT

Thu - MR (CB) Sage

Fri - Problem Session

Mon - CB (RQ)

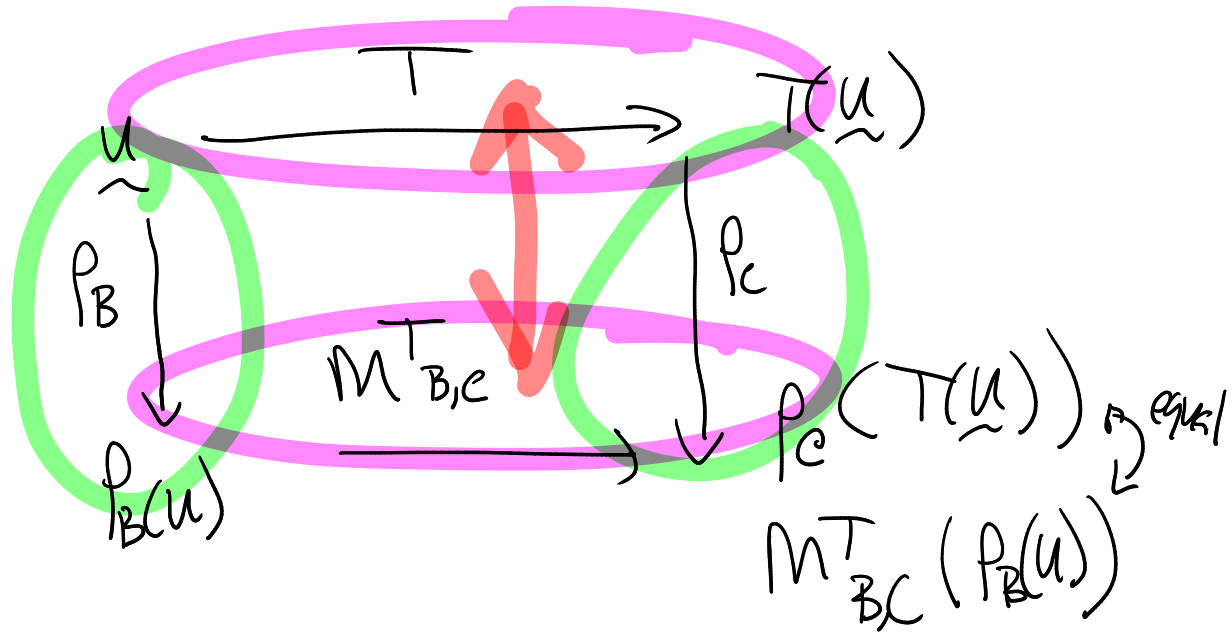
Tue - Problem Session

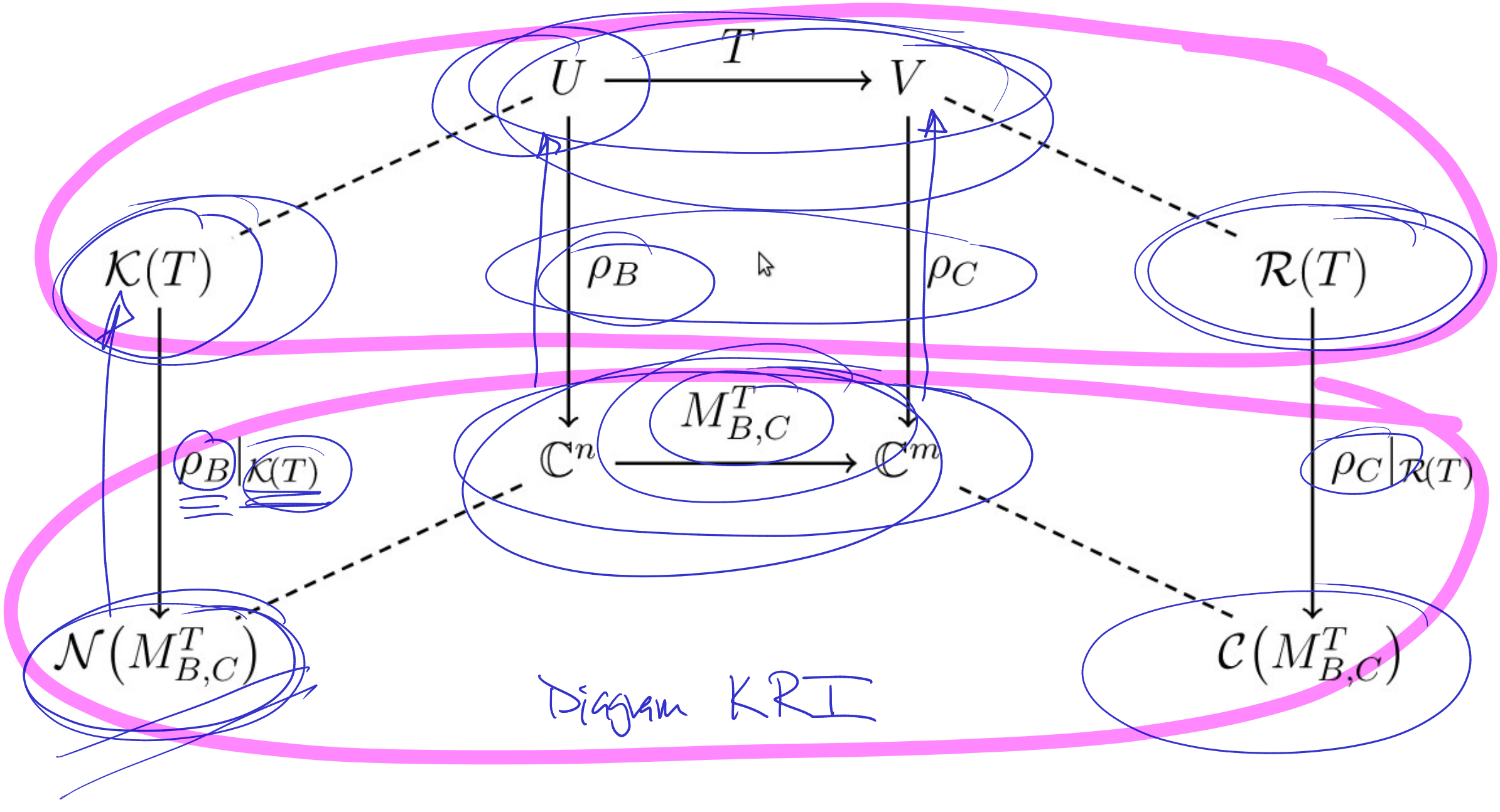
Wed - Exam R

Theorem FTMR $T: U \to V$, $\underset{\sim}{u} \in U$, B, C bases

1) $$\rho_C(T(\underset{\sim}{u})) = M^T_{B,C}\,\rho_B(\underset{\sim}{u})$$

2) $$T(\underset{\sim}{u}) = \rho_C^{-1}\left(M^T_{B,C}\,\rho_B(\underset{\sim}{u})\right)$$




U
T
V
$\mathcal{K}(T)$
ρ_B
ρ_C
$\mathcal{R}(T)$
$M_{B,C}^T$
$\rho_B|_{\mathcal{K}(T)}$
$\mathbb{C}^n$
$\mathbb{C}^m$
$\rho_C|_{\mathcal{R}(T)}$
$\mathcal{N}(M_{B,C}^T)$
$\mathcal{C}(M_{B,C}^T)$
Diagram KRI

Ex $T: M_{22} \to P_2$ $\quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (2a+b+3c-2d) + (5a+3b+7c-4d)x + (a+b+c)x^2$

$C = \left\{ \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 2 & -3 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} -3 & \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix} \right\}$

$E = \{ 1, 1+x, 1+x+x^2 \}$ $\quad {}_{C,E}^{T}M = ?$

$\rho_E\left(T\left(\begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}\right)\right) = \rho_E(5 + 10x + 0x^2) = -5(1) + 10(1+x) + 0(1+x+x^2) = \begin{bmatrix} -5 \\ 10 \\ 0 \end{bmatrix}$

$\rho_E\left(T\left(\begin{bmatrix} & \\ & \end{bmatrix}\right)\right)$ $\quad \rho_E($ $\quad = \begin{bmatrix} -12 \\ 20 \\ 2 \end{bmatrix}$

$\rho_E\left(T\left(\begin{bmatrix} & \\ & \end{bmatrix}\right)\right)$ $\quad = \begin{bmatrix} -5 \\ 12 \\ -1 \end{bmatrix}$

$\rho_E\left(T\left(\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}\right)\right)$ $\quad = \begin{bmatrix} -12 \\ 16 \\ 4 \end{bmatrix}$

${}_{C,E}^{T}M = \begin{bmatrix} -5 & -12 & -5 & -12 \\ 10 & 20 & 12 & 16 \\ 0 & 2 & -1 & 4 \end{bmatrix}$ $\quad 3 \times 4$

$$T\left(\begin{bmatrix}1&2\\3&4\end{bmatrix}\right) = 5+16x+6x^2$$

FTMR

$$= \rho_E^{-1}\left(M_{C,E}^T \rho_C\left(\begin{bmatrix}1&2\\3&4\end{bmatrix}\right)\right) = \rho_E^{-1}\left(M_{C,E}^T \begin{bmatrix}-3\\4\\-2\\-1\end{bmatrix}\right)$$

$$\begin{matrix}15\\-48\\10\\12\end{matrix}$$

$$= \rho_E^{-1}\left(\begin{bmatrix}-11\\10\\6\end{bmatrix}\right) = -11(1) + 10(1+x) + 6(1+x+x^2) = 5+16x+6x^2$$