Math 290A, Friday, April 17 Section ILT

Mon – SLT

Tue – Problem Session

WED — NOTHING

Thu – IVLT

Fri – Problem Session

~~writing~~

Mon – ~~MB~~ VR

writing

SLT & ILT are similar

cut /paste but different

<u>Defn</u> $T : U \to V$, then

T <u>injective</u> if whenever

$T(\underset{\sim}{u_1}) = T(\underset{\sim}{u_2})$ then $\underset{\sim}{u_1} = \underset{\sim}{u_2}$.

Ex $f: \mathbb{R} \to \mathbb{R}$ $f(x) = 5x$ injective

$$f(x_1) = f(x_2)$$
$$5x_1 = 5x_2$$
$$\frac{1}{5}(5x_1) = \frac{1}{5}(5x_2)$$
$$1x_1 = 1x_2$$
$$x_1 = x_2$$

Ex $f(x) = \ln(x)$ injective

$$f(x_1) = f(x_2)$$
$$\ln(x_1) = \ln(x_2)$$
$$e^{\ln(x_1)} = e^{\ln(x_2)}$$
$$x_1 = x_2$$

Ex $g: \mathbb{R} \to \mathbb{R}$ $g(x) = x^2$

not injective

$$g(x_1) = g(x_2)$$

$x_1^2 = x_2^2 \underset{?}{\Longrightarrow} x_1 = x_2$ No

$(-3)^2 = 3^2$, yet $-3 \neq 3$

"1-1"

Ex $T : \mathbb{C}^3 \to \mathbb{C}^4$ $\quad T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} -5x_1 + 4x_2 - 6x_3 \\ -6x_1 + 5x_2 - 7x_3 \\ -x_1 + x_2 - x_3 \\ 3x_1 - 2x_2 + 4x_3 \end{bmatrix}$ is a L.T.

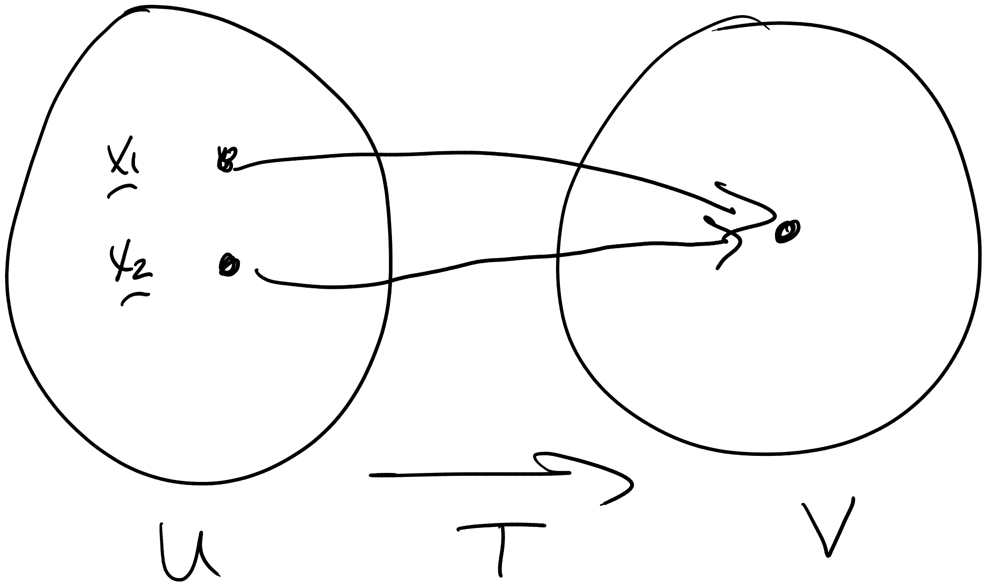
$$\underset{\sim}{x_1} \rightarrow \quad T\left(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} -30 \\ -35 \\ -5 \\ 20 \end{bmatrix}$$

$$\underset{\sim}{x_2} \rightarrow \quad T\left(\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} -30 \\ -35 \\ -5 \\ 20 \end{bmatrix}$$

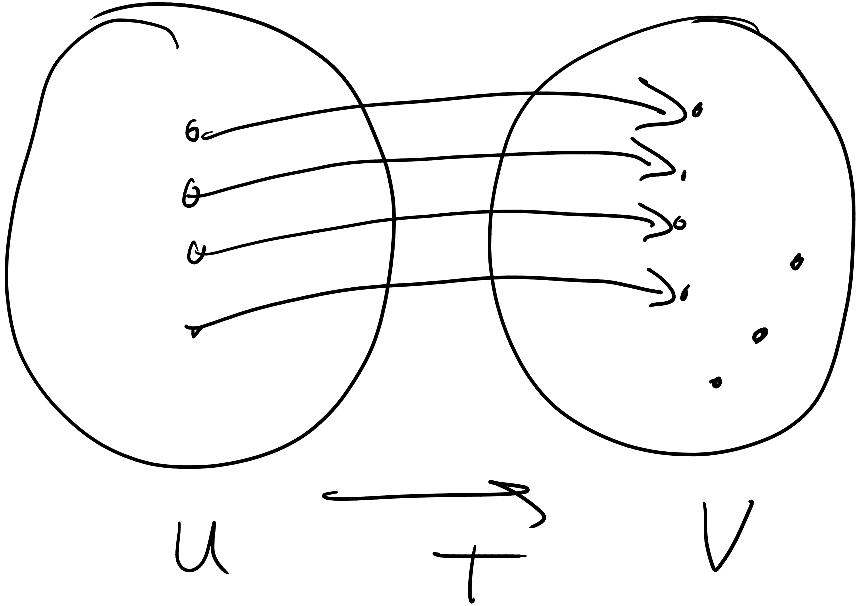
$T(\underset{\sim}{x_1}) = T(\underset{\sim}{x_2})$ yet $\underset{\sim}{x_1} \neq \underset{\sim}{x_2}$

So T is *not* injective

not injective



injective



Defn $T: U \to V$ define the kernel of T

$$K(T) = \{ \underset{\sim}{u} \in U \mid T(\underset{\sim}{u}) = \underset{\sim}{0} \}$$

Ex $K(T)$ for previous T ↑.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \underset{\sim}{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5x_1 + 4x_2 - 6x_3 \\ -6x_1 + 5x_2 - 7x_3 \\ -x_1 + x_2 - x_3 \\ 3x_1 - 2x_2 + 4x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

homogeneous system

4 equations, 3 vars

coefficient matrix

RREF

$$\rightarrow \begin{bmatrix} \textcircled{1} & 0 & 2 \\ 0 & \textcircled{1} & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Solutions $N(\quad) = \left\langle \left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix} \right\} \right\rangle = K(T)$

Now $T\left(\begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}\right) = T\left(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}\right) + T\left(\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}\right) + \underset{\sim}{0}$

$= T\left(\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}\right)$

Theorem KILT $\quad T$ injective $\iff K(T) = \{\underset{\sim}{0}\}$

Ex $T : P_2 \to M_{22} \quad T(a+bx+cx^2) = \begin{bmatrix} a-2b+c & -a+3b \\ 2b+3c & a-5b-4c \end{bmatrix}$

$K(T)$?

$T(a+bx+cx^2) = \underset{\sim}{0}$

$$\begin{bmatrix} a-2b+c & -a+3b \\ 2b+3c & a-5b-4c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

homogeneous system
4 equations, 3
variables,
RREF coefficient matrix

RREF $\longrightarrow \begin{bmatrix} \textcircled{1} & 0 & 0 \\ 0 & \textcircled{1} & 0 \\ 0 & 0 & \textcircled{1} \\ 0 & 0 & 0 \end{bmatrix}$

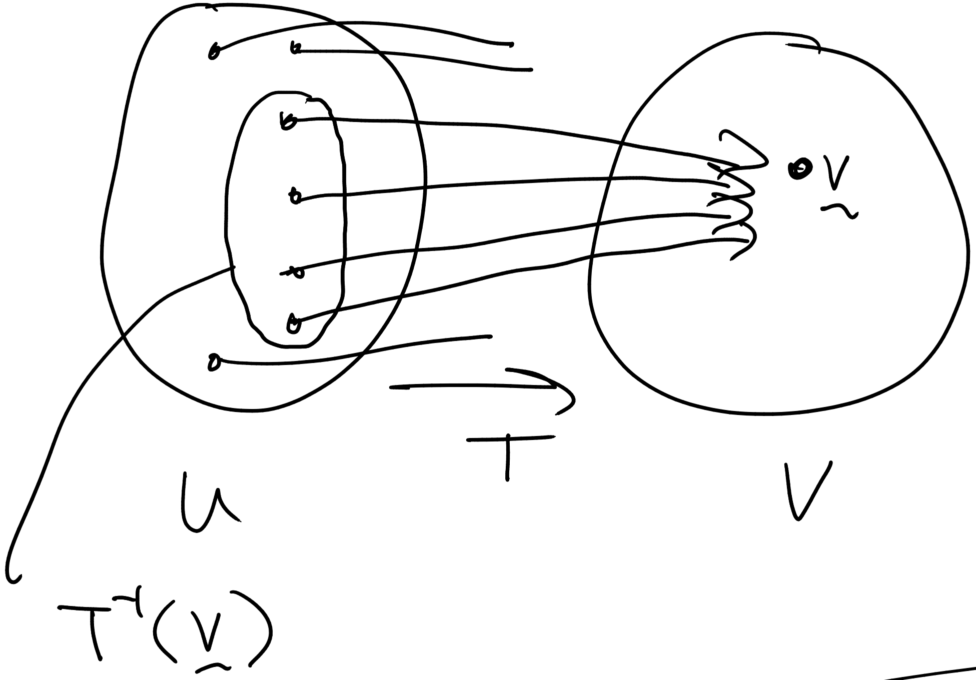
So $a = b = c = 0 \qquad$ So if $a+bx+cx^2 \in K(T)$ then $a+bx+cx^2 = 0+0x+0x^2$

$K(T) = \{0+0x+0x^2\} = \{\underset{\sim}{0}\} \quad \Rightarrow T$ injective.

Theorem $\quad K(T)$ subspace (of U).

Defn $T: U \to V$, $\underset{\sim}{v} \in V$, $T^{-1}(\underset{\sim}{v}) = \{\underset{\sim}{u} \in U \mid T(\underset{\sim}{u}) = \underset{\sim}{v}\}$

$\uparrow$ pre-image of $\underset{\sim}{v}$.



Theorem KPI

Suppose $T(\underset{\sim}{u}) = \underset{\sim}{v}$

then $T^{-1}(\underset{\sim}{v}) = \underset{\sim}{u} \oplus K(T)$

$= \{\underset{\sim}{u} + \underset{\sim}{z} \mid \underset{\sim}{z} \in K(T)\}$