Math 290A, Monday, April 13 Section SD

Mon - SD
Sage

Tue - Problem Session
Writing D&E - 2 - beezer.pdf
Bring Your Pet to Class

Wed - Exam D&E

Thu - LT RQ

Fri - ILT
W/WF
CR/NC

<u>Similarity</u> A, B square

A & B similar if there exists S so that $S^{-1}AS = B$

<u>Equivalence Relation</u>

<u>Theorem</u> A & B similar $\Longrightarrow$ A & B have identical eigenvalues

<u>Proof</u> Know there is S so that $S^{-1}AS = B$

$$p_B(\lambda) = \det(B - \lambda I) = \det(S^{-1}AS - \lambda S^{-1}S)$$

$$= \det(S^{-1}(A - \lambda I)S) = \det(S^{-1})\,\det(\underline{A - \lambda I})\,\det(S)$$

$$= \det(S^{-1})\det(S)\det(A - \lambda I) = \det(S^{-1}S)\; p_A(\lambda)$$

$$= \det(I)\, p_A(\lambda) = 1 \cdot p_A(\lambda) = p_A(\lambda)$$

Same roots (eigenvalues) of equal characteristic polynomials.

NOT THE CONVERSE

Ex $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ A, B have same eigenvalues, $\lambda = 1, 1$

Are the matrices similar? NO. Try

$$B = S^{-1} A S = S^{-1} I_2 S = S^{-1} S = I_2 \;\not\Rightarrow$$

Diagonalization

Given A is there a diagonal matrix D and non singular S so that $S^{-1} A S = D$?

Sometimes If so D has eigenvalues of A on diagonal.

And S is a matrix w/ eigenvectors of A as its columns.

Theorem DC $\underset{n\times n}{A}$ is <u>diagonalizable</u> $\Longleftrightarrow$ there is a set of n linearly independent eigenvectors of A.

Proof ($\Leftarrow$)

Let $T = \{\underset{\sim}{x_1}, \underset{\sim}{x_2}, \ldots, \underset{\sim}{x_n}\}$ eigenvectors for A w/ eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$; linearly independent

Form $S = [\underset{\sim}{x_1} | \underset{\sim}{x_2} | \cdots \underset{\sim}{x_n}]$

$$D = \begin{bmatrix} \lambda_1 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_n \end{bmatrix} = [\lambda_1 \underset{\sim}{e_1} | \lambda_2 \underset{\sim}{e_2} | \cdots | \lambda_n \underset{\sim}{e_n}]$$

$$\begin{aligned} AS &= A[\underset{\sim}{x_1} | \underset{\sim}{x_2} | \cdots | \underset{\sim}{x_n}] = [A\underset{\sim}{x_1} | A\underset{\sim}{x_2} | \cdots | A\underset{\sim}{x_n}] \\ &= [\lambda_1 \underset{\sim}{x_1} | \lambda_2 \underset{\sim}{x_2} | \cdots \lambda_n \underset{\sim}{x_n}] = [\lambda_1 S\underset{\sim}{e_1} | \lambda_2 S\underset{\sim}{e_2} | \cdots | \lambda_n S\underset{\sim}{e_n}] \\ &= [S(\lambda_1 \underset{\sim}{e_1}) | S(\lambda_2 \underset{\sim}{e_2}) | \cdots | S(\lambda_n \underset{\sim}{e_n})] = S[\lambda_1 \underset{\sim}{e_1} | \lambda_2 \underset{\sim}{e_2} | \cdots | \lambda_n \underset{\sim}{e_n}] = SD \end{aligned}$$

S nonsingular so $AS = SD \longrightarrow S^{-1}AS = D$