Math 290A, Friday, April 10, Section PEE

Mon - SD

Tue - Problem Session

Writing D&E

Bring your Pet to Class Day

Wed - Exam D&E

Office Hours: Regular Times, Pacific

Google Meet

Theorem EDELI $S = \{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_p\}$ eigenvectors of A for eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_p$, with $\lambda_i \neq \lambda_j$. Then S is linearly independent.

Proof By contradiction, assume S linearly dependent.

Index k so that $\{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_{k-1}\}$ linearly independent set

$\{\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_{k-1}, \underset{\sim}{x}_k\}$ linearly dependent.

there are scalars, not all zero, so that

$$\underset{\sim}{0} = a_1\underset{\sim}{x}_1 + a_2\underset{\sim}{x}_2 + \dots + a_k\underset{\sim}{x}_k$$

① $\underset{\sim}{0} = \lambda_k \underset{\sim}{0} = \lambda_k(a_1\underset{\sim}{x}_1 + a_2\underset{\sim}{x}_2 + \dots + a_k\underset{\sim}{x}_k) = a_1\lambda_k\underset{\sim}{x}_1 + a_2\lambda_k\underset{\sim}{x}_2 + \dots \boxed{+ a_k\lambda_k\underset{\sim}{x}_k}$

② $\underset{\sim}{0} = A\underset{\sim}{0} = A(a_1\underset{\sim}{x}_1 + a_2\underset{\sim}{x}_2 + \dots + a_k\underset{\sim}{x}_k)$

$= a_1 A\underset{\sim}{x}_1 + a_2 A\underset{\sim}{x}_2 + \dots + a_k A\underset{\sim}{x}_k$ MMDAA

$= a_1\lambda_1\underset{\sim}{x}_1 + a_2\lambda_2\underset{\sim}{x}_2 + \dots + \boxed{a_k\lambda_k\underset{\sim}{x}_k}$

" Subtract ② from ①

$\underset{\sim}{0} - \underset{\sim}{0} = a_1\lambda_k \underset{\sim}{x}_1 - a_1\lambda_1\underset{\sim}{x}_1 + a_2\lambda_k \underset{\sim}{x}_2 - a_2\lambda_2\underset{\sim}{x}_2 + \dots + a_{k-1}\lambda_k\underset{\sim}{x}_{k-1} - a_{k-1}\lambda_{k-1}\underset{\sim}{x}_{k-1}$

$\underset{\sim}{0} = a_1(\lambda_k - \lambda_1)\underset{\sim}{x}_1 + a_2(\lambda_k - \lambda_2)\underset{\sim}{x}_2 + \dots + a_{k-1}(\lambda_k - \lambda_{k-1})\underset{\sim}{x}_{k-1}$

This is an RLD on a LI set. Thus scalars are all zero.

So $a_1\underbrace{(\lambda_k - \lambda_1)}_{\text{nonzero}} = 0$, $a_2\underbrace{(\lambda_k - \lambda_2)}_{\text{nonzero}} = 0$, ..., $a_{k-1}\underbrace{(\lambda_k - \lambda_{k-1})}_{\text{nonzero}} = 0$

So $a_1 = 0$, $a_2 = 0$, ..., $a_{k-1} = 0$ This contradicts the a_i's definition.

Theorem HMRE A Hermitian matrix $\Rightarrow$ all eigenvalues real.

Proof Let $\underset{\sim}{x}$ be an eigenvector of A for λ.

$\lambda\langle \underset{\sim}{x}, \underset{\sim}{x}\rangle = \langle \underset{\sim}{x}, \lambda\underset{\sim}{x}\rangle = \langle \underset{\sim}{x}, A\underset{\sim}{x}\rangle \overset{\text{A Hermitian}}{=} \langle A\underset{\sim}{x}, \underset{\sim}{x}\rangle = \langle \lambda\underset{\sim}{x}, \underset{\sim}{x}\rangle = \bar{\lambda}\underbrace{\langle \underset{\sim}{x}, \underset{\sim}{x}\rangle}_{\neq 0}$

Theorem PIP $\underset{\sim}{x} \neq \underset{\sim}{0}$

"Cancel" $\langle \underset{\sim}{x}, \underset{\sim}{x}\rangle$

get $\lambda = \bar{\lambda} \Rightarrow \lambda \in \mathbb{R}$