Math 290, Friday, April 3 Section DM

Mon - Problem Session (Chapter VS)
Writing 10 AM Pacific

Tue - PDM
Sage

Wed - Exam VS

$$\begin{bmatrix} + & - & + & - \\ - & + & - & + \\ + & - & + & - \\ - & + & - & + \end{bmatrix}$$

Determinant is a function from a square matrix to a number.

\det

Ex

$$A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 5 & 0 & 0 & 3 \\ 2 & 6 & 1 & -4 \\ 1 & 0 & 3 & 1 \end{bmatrix}$$

expansion about row 2

$\det(A) = |A|$

$$= 5(-1)\begin{vmatrix} -1 & 3 & 4 \\ 6 & 1 & -4 \\ 0 & 3 & 1 \end{vmatrix} + 0(1)\begin{vmatrix} \; \end{vmatrix} + 0(-1)\begin{vmatrix} \; \end{vmatrix} + 3(1)\begin{vmatrix} 2 & -1 & 3 \\ 2 & 6 & 1 \\ 1 & 0 & 3 \end{vmatrix}$$

$$= -5\left(0\left|\cdots\right| - 3\begin{vmatrix} -1 & 4 \\ 6 & -4 \end{vmatrix} + 1\begin{vmatrix} -1 & 3 \\ 6 & 1 \end{vmatrix}\right) + 3\left(1\begin{vmatrix} -1 & 3 \\ 6 & 1 \end{vmatrix} - 0\left|\cdots\right| + 3\begin{vmatrix} 2 & -1 \\ 2 & 6 \end{vmatrix}\right)$$

$$= -5(60 - 19) + 3(-19 + 42) = -205 + 69 = -236$$

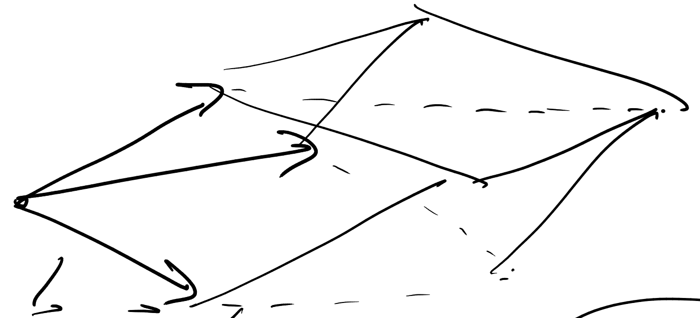
Sage: det(A)

Ex

$$B = \begin{bmatrix} 2 & 3 & 6 & -1 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

$$|B| = 2\left|\quad\right| = 2\cdot 5\left|\quad\right| = 2\cdot 5(-2\cdot 8)$$

$$= -160$$



$2\vec{i} + 3\vec{j} + 5\vec{k}$

$$\vec{u}\cdot(\vec{v}\times\vec{w}) = \begin{vmatrix} 2 & 3 & 5 \\ \sim & & \\ \sim & & \end{vmatrix}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & x & x \\ x & x & x \end{vmatrix}$$

Elementary Matrices

<u>Ex</u> $E_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $A = \begin{bmatrix} 2 & -3 & 4 & 6 \\ 9 & 5 & -2 & 3 \\ 6 & 1 & 4 & 0 \\ 5 & 2 & 3 & -1 \end{bmatrix}$

$E_{23}A = \begin{bmatrix} 2 & -3 & 4 & 6 \\ 6 & 1 & 4 & 0 \\ 9 & 5 & -2 & 3 \\ 5 & 2 & 3 & -1 \end{bmatrix} = B$ $\leftarrow$ $R_2 \leftrightarrow R_3$

matrix product

swap row operation

$\det(E_{23}) = -1$

<u>Fact</u> $\det(A) = -\det(B)$

Properties of Determinants

1) $\det(A) = \det(A^t)$

2) If A has a row of all zeros (or a column of all zeros) then $\det(A) = 0$.

3) If A has two identical columns (or two identical rows) then $\det(A) = 0$

4) If A & B differ by a column swap (or a row swap) then $\det(A) = -\det(B)$

5) Suppose B is ob ~~tained~~ from A by multiplying a column (or row) by a scalar α. Then $\det(B) = \alpha \det(A)$

6) Suppose B is obtained from A by multiplying a column (or row) by α & adding it to another.
Then $\det(B) = \det(A)$