Math 290, Friday, April 3  Section DM
Mon - Problem Session (Chapter VS)
      Writing 10 am Pacific

Tue - PDM
      Sage

Wed - Exam VS

Determinant is a function from a square matrix to a number.

Ex
\[ A = \begin{bmatrix} 2 & -1 & 3 & 4 \\ 5 & 0 & 0 & 3 \\ 2 & 6 & 1 & -4 \\ 1 & 0 & 3 & 1 \end{bmatrix} \]

\[ \text{det} (A) = 1A \]

\[ = 5(4) \begin{vmatrix} -1 & 3 \\ 0 & 3 \end{vmatrix} + 0(1) \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} + 3(1) \begin{vmatrix} 2 & -1 & 4 \\ 1 & 0 & 3 \end{vmatrix} \]

\[ = 5(4)\begin{vmatrix} -1 & 3 \\ 0 & 3 \end{vmatrix} = 5(4)(-3) = -60 \]
\[ = -5 \begin{vmatrix} 0 & 1 & -3 \\ 6 & -4 & 1 \\ 0 & 1 & 3 \end{vmatrix} + 3 \begin{vmatrix} 1 & 1 & 3 \\ 6 & 1 & 0 \\ 0 & 3 & 2 \end{vmatrix} \]

\[ = -5 \begin{vmatrix} 60 & -19 \\ -19 & 42 \end{vmatrix} = -205 + 69 = -236 \]

\[ \text{Sage: } \det(A) \]

\[ \det(B) = \begin{vmatrix} 2 & 3 & 6 & -1 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -5 \end{vmatrix} \]

\[ = 2 \begin{vmatrix} 3 & 6 & -1 \\ 5 & 2 & 3 \\ 0 & -2 & 1 \end{vmatrix} \]

\[ = 25 \begin{vmatrix} 3 & 6 & -1 \\ 5 & 2 & 3 \\ 0 & -2 & 1 \end{vmatrix} \]

\[ = 25 \begin{vmatrix} -14 & -4 \\ 4 & -1 \end{vmatrix} \]

\[ = -160 \]

\[ \vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 3 & 5 & 6 \end{vmatrix} \]
Elementary Matrices

\[ E_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ A = \begin{bmatrix} 2 & -3 & 4 & 6 \\ 9 & 5 & -2 & 3 \\ 6 & 1 & 4 & 0 \\ 5 & 2 & 3 & -1 \end{bmatrix} \]

\[ E_{23} A = \begin{bmatrix} 2 & -3 & 4 & 6 \\ 6 & 1 & 4 & 0 \\ 9 & 5 & -2 & 3 \\ 5 & 2 & 3 & -1 \end{bmatrix} \]

\[ \text{Matrix product} \]

\[ 2 \rightarrow \text{R}_3 \text{ swap row operation} \]

\[ \det(E_{23}) = -1 \]

\[ \text{Fact} \quad \det(A) = - \det(B) \]
Properties of Determinants

1) \( \det (A) = \det (A^t) \)

2) If \( A \) has a row of all zeros (or a column of all zeros) then \( \det (A) = 0 \).

3) If \( A \) has two identical columns (or two identical rows) then \( \det (A) = 0 \).

4) If \( A \) & \( B \) differ by a column swap (or a row swap) then \( \det (A) = -\det (B) \).

5) Suppose \( B \) is obtained from \( A \) by multiplying a column by a scalar \( \alpha \). Then \( \det (B) = \alpha \det (A) \) (or row).
6) Suppose $B$ is obtained from $A$ by multiplying a column (or row) by $\alpha$ and adding it to another. Then $\det(B) = \det(A)$. 